KPP Implementation in ROMS

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- Overview and history

- Quasi-physical instabilities
  1. Ri_g instability
  2. Chequer-board instability

- Sources of temporal discontinuous behavior
  1. vertical grid-point locking
  2. Hysteresis h_{MO} limitation logic
  3. Hysteresis h_{Ek} limitation logic

- Integral Ri_b formulation

- Ekman depth limitation inserted in Ri_b

- Options for vertical discretization

- Comparisons, practices, etc

- Unsettled issues

KPP boundary layer model: An Overview

Extent of HBL h_{bl} is set by Richardson number, LMD94,

$$Ri_b(z) = \frac{g\left[\rho_r - \rho(z)\right]/\rho_0}{\left|\mathbf{V}_r - \mathbf{V}(z)\right|^2/\Delta z + V_t^2(z)/\Delta z}$$

$$Ri_b(-h_{\mathrm{bl}}) = Ri_{\mathrm{cr}} = 0.3$$

then HBL is limited by

- Monin-Obukhov depth $h^{\mathrm{MO}} = \frac{C^{\mathrm{MO}} \cdot u_*^3}{\kappa \cdot Bf}$

- Ekman depth $h_{\mathrm{Ek}} = 0.7 u_*/f$

Once h_{bl} is determined, mixing coefficients within PBL are recomputed using cubic fit, set by forcing conditions at surface and matching mixing coefficients just below PBL.

$Ri_b(z)$ disregards velocity profile and 3D-nality within PBL

$Ri_b(z)$ oscillates if $\mathbf{V}(z)$ is Ekman spiral (prevented by h_{Ek} only)

Summary of changes in KPP since 1994 by W. Large and G. Danabasoglu (2003).

- Turbulent velocity scale limit in **stable** regions
- Diurnal cycle in SW Rad. heat flux
- Critical bulk Ri depends on vertical resolution
- C_v depends on BVF
- Correct Ekman and Monin-Obukhov depth limit computations
- Compute interior convection **after** BL mixing is done
- Modify usage of N in turbulent shear computation
- Quadratic interpolation of Ri to find h_{bl}

Monin-Obukhov depth

$$h^{\mathsf{MO}} = \frac{C^{\mathsf{MO}} \cdot u_*^3}{\kappa \cdot Bf}$$

where, due to solar radiation absorption $Bf = Bf(z)$ increases from the surface downward, possibly changing sign, and

$Bf(z) > 0$ limiting $\qquad$ $Bf(z) > 0$ not limiting

$\Rightarrow$ possibility $Bf\,(-\text{overestimated } h_{\mathsf{bl}}) > 0$, but $Bf\left(-h^{\mathsf{MO}}\right) < 0$

solution: use $Bf = Bf\left(-h^{\mathsf{MO}}\right)$ in setting h^{MO}, i.e. implicit search for k embracing z^*,

$$z_k \le z^* \le z_{k+1} \quad : \qquad h^{\mathsf{MO}}(z_k) \le |z^*| \le h^{\mathsf{MO}}\left(z_{k+1}\right)$$

then set

$$\frac{{h^{\mathsf{MO}}}_k\left(z_{k+1} - z^*\right) + {h^{\mathsf{MO}}}_{k+1}\left(z^* - z_k\right)}{z_{k+1} - z_k} + z = 0$$

Monin-Obukhov depth continued...

resulting

$$z = -\frac{\frac{C^{\mathrm{MO}} u_*^3}{\kappa}\left(Bf'_{k+1} z_{k+1} - Bf'_k z_k\right)}{Bf'_{k+1} Bf'_k \left(z_{k+1} - z_k\right) + \frac{C^{\mathrm{MO}} u_*^3}{\kappa}\left(Bf'_k - Bf'_{k+1}\right)} \Rightarrow -h^{\mathrm{MO}}$$

(just above $Bf' = \max(Bf, 0)$; if k not found $\Rightarrow$ no limit.)

- no singularity if either $Bf \to 0$
- h_{bl} is not involved $\Rightarrow$ no hysteresis
- limitation applied outside $Bf > 0$ logic: is already taken into account

Alternative (integral) criterion for h_{bl}

Richardson number criterion defines boundary layer as the hight of water column within which turbulent shear production is balanced by dissipation due to stratification:

$\Rightarrow$ construct

$$Cr(z) = \int_{z}^{\text{surf}} \left\{ \left| \frac{\partial \mathbf{V}}{\partial z} \right|^2 + \frac{1}{Ri_c} \cdot \frac{g}{\rho_0} \cdot \left. \frac{\partial \rho}{\partial z} \right|_{\text{ad}} \right\} dz' + \frac{V_t^2(z)}{z}$$

and search for crossing point $Cr(z) = 0$.

- Same result as Ri_b the case of linear velocity profile, but otherwise

$$\int_{z'}^{z''} \left| \frac{\partial \mathbf{V}}{\partial z} \right|^2 dz \geq \frac{\left| \mathbf{V}'' - \mathbf{V}' \right|^2}{z'' - z'}$$

- $Cr(z)$ is *monotonic* for Ekman spiral

- Avoids introduction of *reference* potential density: basically integration Brunt-Väisäla frequency

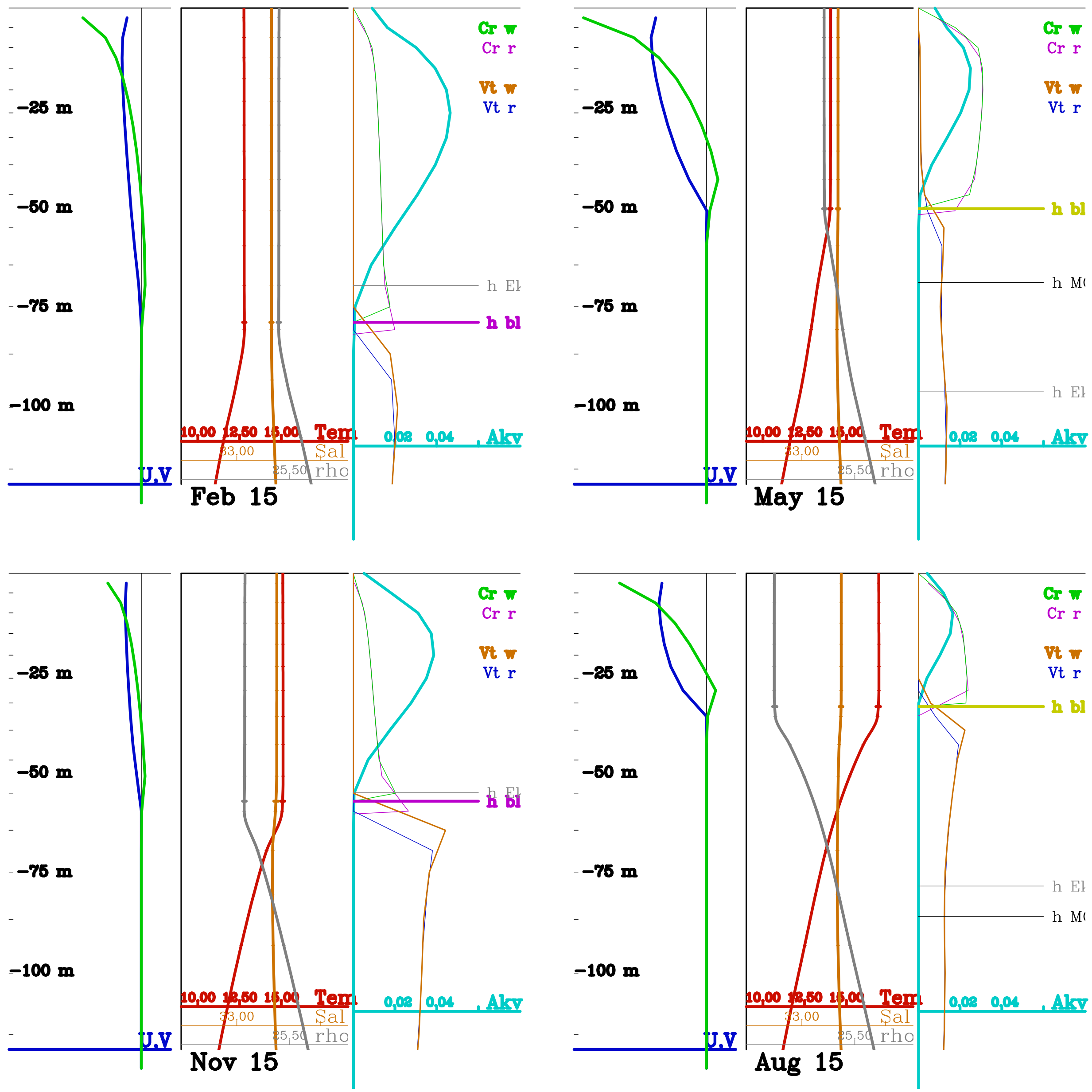
- Formalism of *adiabatic* derivatives and differences

- Correct account for thermobaric effect: for well mixed layer it is equivalent to bringing fluid parcel from surface *to the edge of BL* and comparing with ambient fluid *there*

- Numerically more attractive, since $\mathbf{V}(z)$ and $\rho(z)$ can be reconstructed as continuous functions

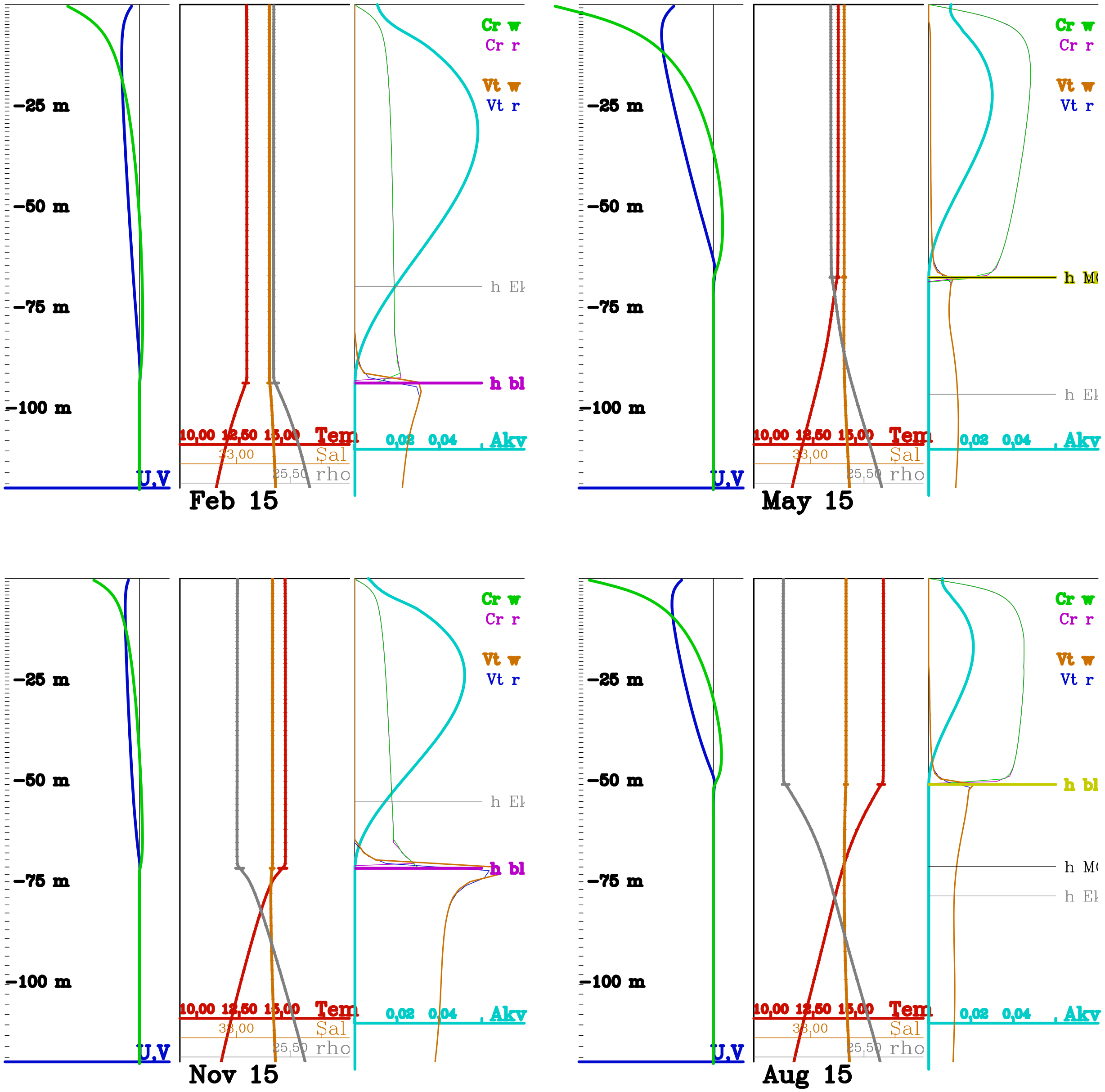
- Combines with computation of *local* (gradient) Richardson number

- Avoids ambiguity for merging top and bottom BL



-25 m
-50 m
-75 m
-100 m
U,V
10,00 12,50 15,00 Tem
33,00 Sal
25,50 rho
0,02 0,04 Akv
Cr w
Cr r
Vt w
Vt r
h bl
h Ek
h MO
Feb 15
May 15
Nov 15
Aug 15

N=40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N=240

Ekman depth $h_{\mathsf{Ek}} = 0.7u_*/f$

Length u_*/f and velocity scale u_* are natural scaling parameters for neutrally stratified problem

$$\frac{\partial}{\partial z}\left(w_*|z|\frac{\partial \mathbf{v}}{\partial z}\right) = -if\mathbf{v}$$

where $\mathbf{v} = u + iv$, and $w_* = \kappa u_*$, and κ is von Karman constant.

??? rather than imposing h_{Ek} as hard constraint, "teach" KPP to handle Ekman boundary layer

Modified Ekman problem

$$\frac{\partial}{\partial z}\left[w_* L G\left(\frac{z}{L}\right)\frac{\partial \mathbf{v}}{\partial z}\right] = -if\mathbf{v}$$

where G is a non-dimensional shape function

$$G(\sigma) = |\sigma|\,(1-\sigma)^2 + \begin{cases} \frac{(\sigma-\sigma_0)^2}{2\sigma_0}, & \sigma < \sigma_0 \\ 0 & \text{otherwise} \end{cases}$$

$\sigma_0 = 0.1$

$$w_* L G(0)\left.\frac{\partial \mathbf{v}}{\partial z}\right|_{z=0} = u_*^2 \qquad \left.\frac{\partial \mathbf{v}}{\partial z}\right|_{z=0} = \frac{u_*}{\kappa L \sigma_0/2}$$

and $\mathbf{v} = 0$ at $z = -L$.

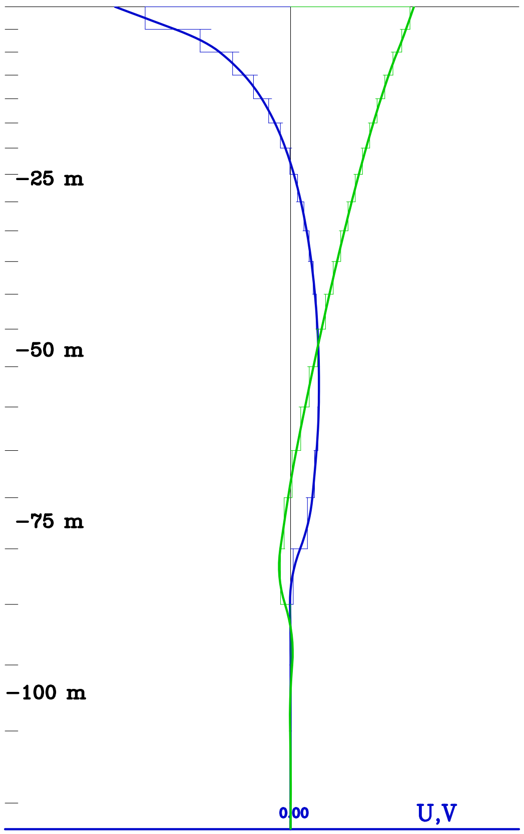
Nondimensionalization: Postulate that depth of generated this way boundary layer is equal to Ekman length and introduce scaling

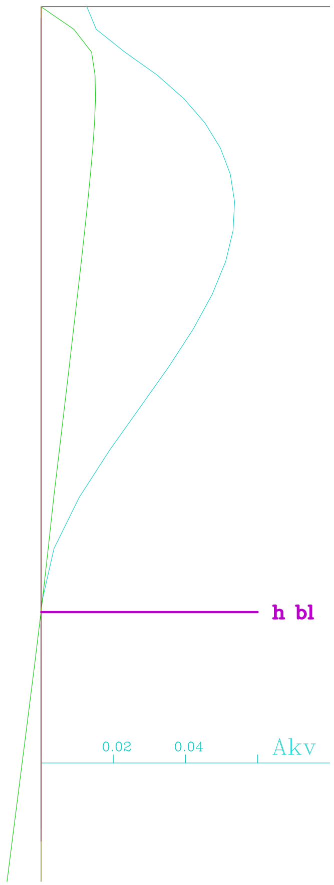
$$z = L\sigma = \sigma \cdot 0.7 u_* / f \qquad\qquad \mathbf{v} = u_* \cdot \tilde{\mathbf{v}}$$

resulting

$$\frac{\partial}{\partial\sigma}\left(G(\sigma)\frac{\partial\tilde{\mathbf{v}}}{\partial\sigma}\right) = -i\frac{\kappa}{0.7}\tilde{\mathbf{v}} \qquad\qquad \left.\frac{\partial\tilde{\mathbf{v}}}{\partial\sigma}\right|_{\sigma=0} = \frac{2}{\kappa\sigma_0},$$

where everything has been scaled out.





Recognize Coriolis force as *stabilizing* effect (balancing vertical shear production), construct

$$Cr(z) = \int_z^{\text{surf}} \left\{ \left| \frac{\partial \mathbf{v}}{\partial z} \right|^2 - C_{\mathsf{Ek}} \cdot f^2 \right\} dz'$$

and apply the same scaling

$$\widetilde{Cr}(\sigma) = \frac{1}{(0.7)^2} \int_\sigma^0 \left| \frac{\partial \tilde{\mathbf{v}}}{\partial \sigma} \right|^2 d\sigma' - C_{\mathsf{Ek}} \cdot |\sigma|$$

and demand that $\widetilde{Cr}(-1) = 0$.

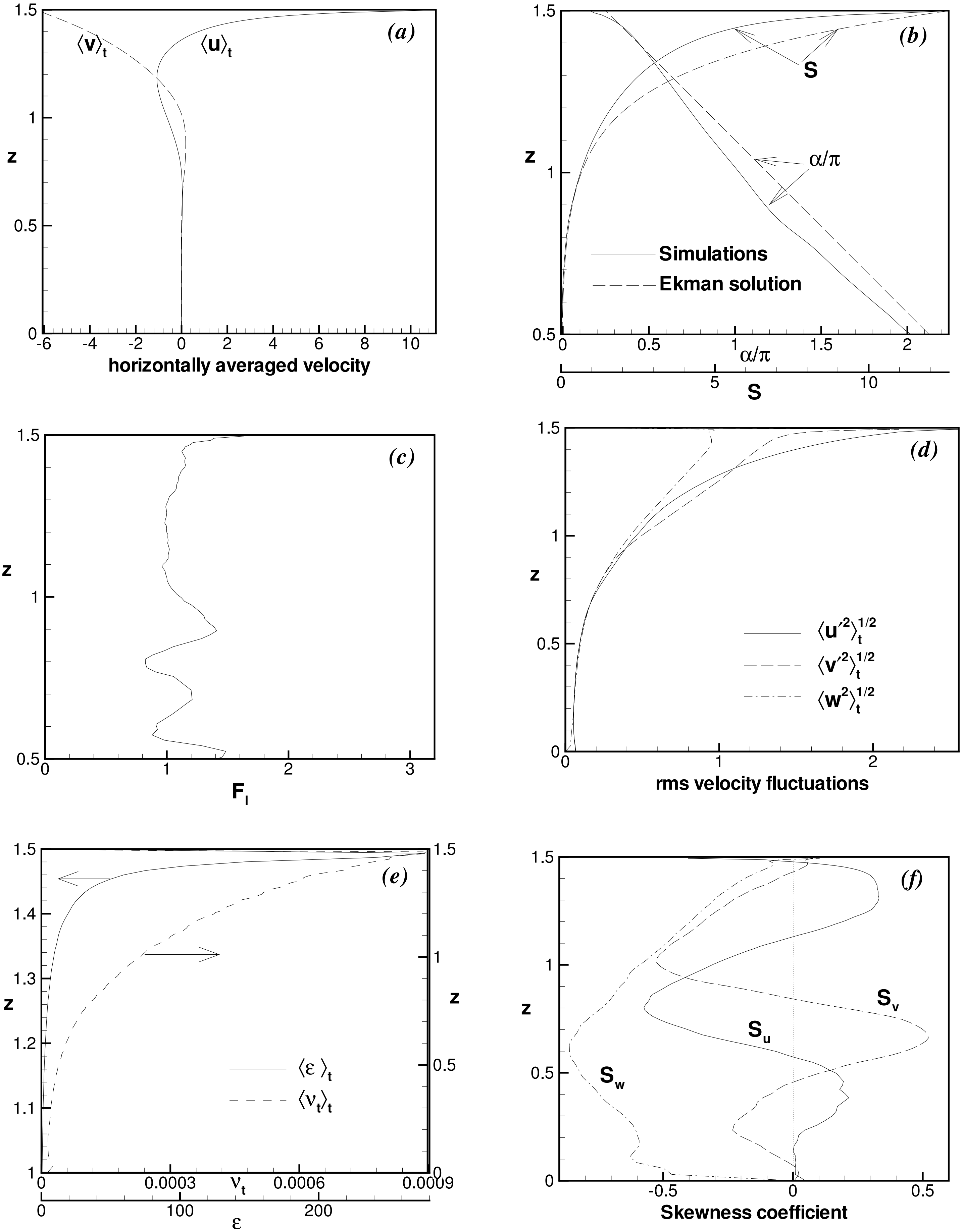
$C_{\mathsf{Ek}} \approx 400$

Verify against DNS and LES simulations:

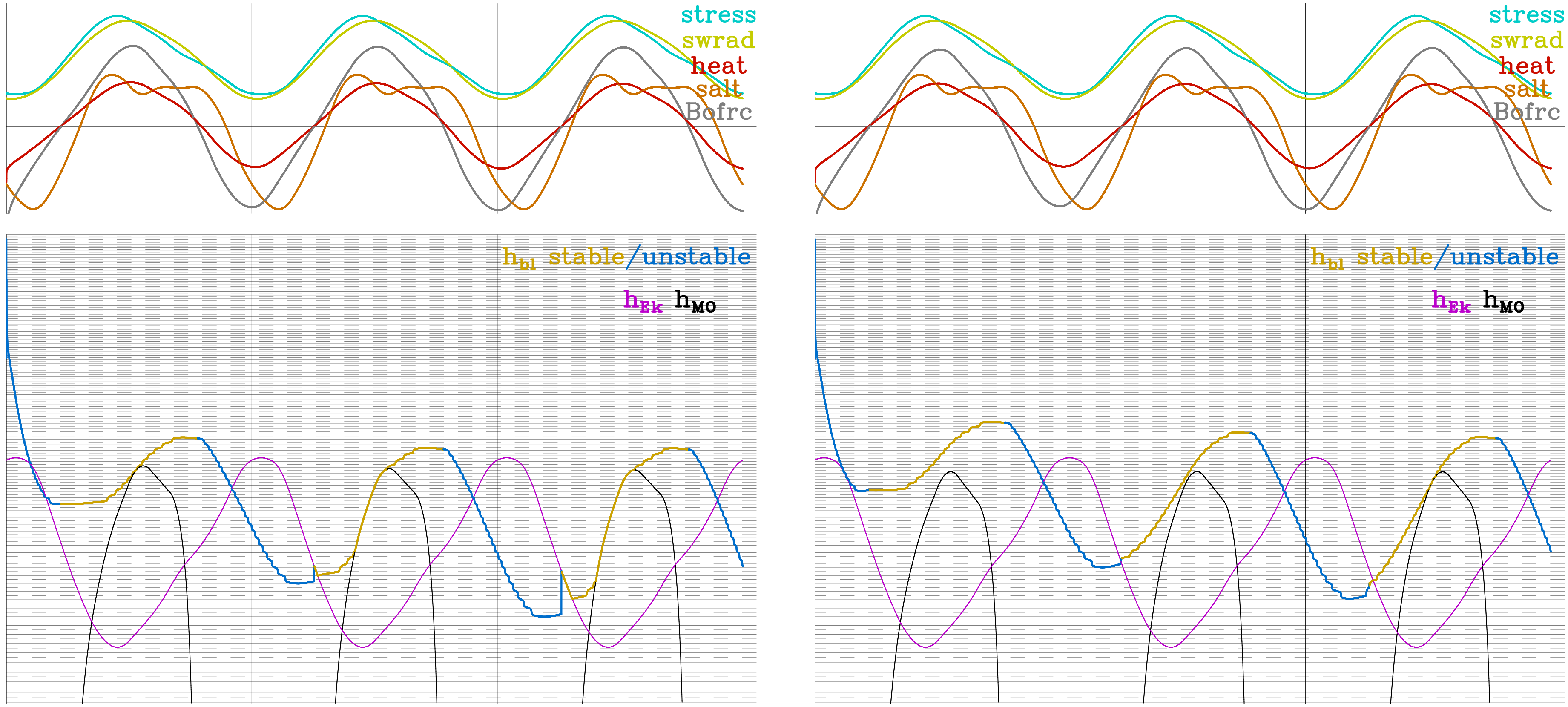
- Coleman G., 1999 Similarity statistics from direct numerical simulation of the neutrally stratified PBL. *JAS*, **56**, 891-900.

- Zikanov, 0., D. N. Slinn, and M. R. Dhanak, 2003, Large-eddy simulation of the wind-induced turbulent Ekman layer. *JFM*

??? Opposite regime: rotating convection?



(a)
⟨v⟩t
⟨u⟩t
z
horizontally averaged velocity
(b)
S
α/π
Simulations
Ekman solution
z
α/π
S
(c)
z
F_l
(d)
⟨u'²⟩t^1/2
⟨v'²⟩t^1/2
⟨w²⟩t^1/2
z
rms velocity fluctuations
(e)
⟨ε ⟩t
⟨νt⟩t
z
νt
ε
(f)
Sv
Su
Sw
z
Skewness coefficient



Ekman, Monin-Obukhov depth and h_{bl} from KPP model under slowly varying (seasonal) forcing: Note abrupt changes in h_{bl} when buoyancy forcing changes from unstable to stable. **Top:** forcing fluxes at surface; **Right:** restriction $h_{\mathsf{bl}} \leq h_{\mathsf{Ek}}$ if $Bofrc > 0$; **Right:** new Ekman length limitation algorithm. Note that h_{Ek} crosses h_{bl} approximately where the regime changes from unstable to stable.

Numerical Issues

- Computation of Ri_b/Cr at vertical ρ vs. W-points

- ρ-placement is natural for finite-difference (trapezoidal-rule) terms in Ri_b/Cr (but not for V_t^2), however

$$A_{k+1/2} \sim \left(z_{k+1/2} - |h_{\text{bl}}|\right)^2$$

  near the edge of PBL, hence needs h_{bl} needs accuracy relatively to W-points, while missing ρ-s is more forgiving

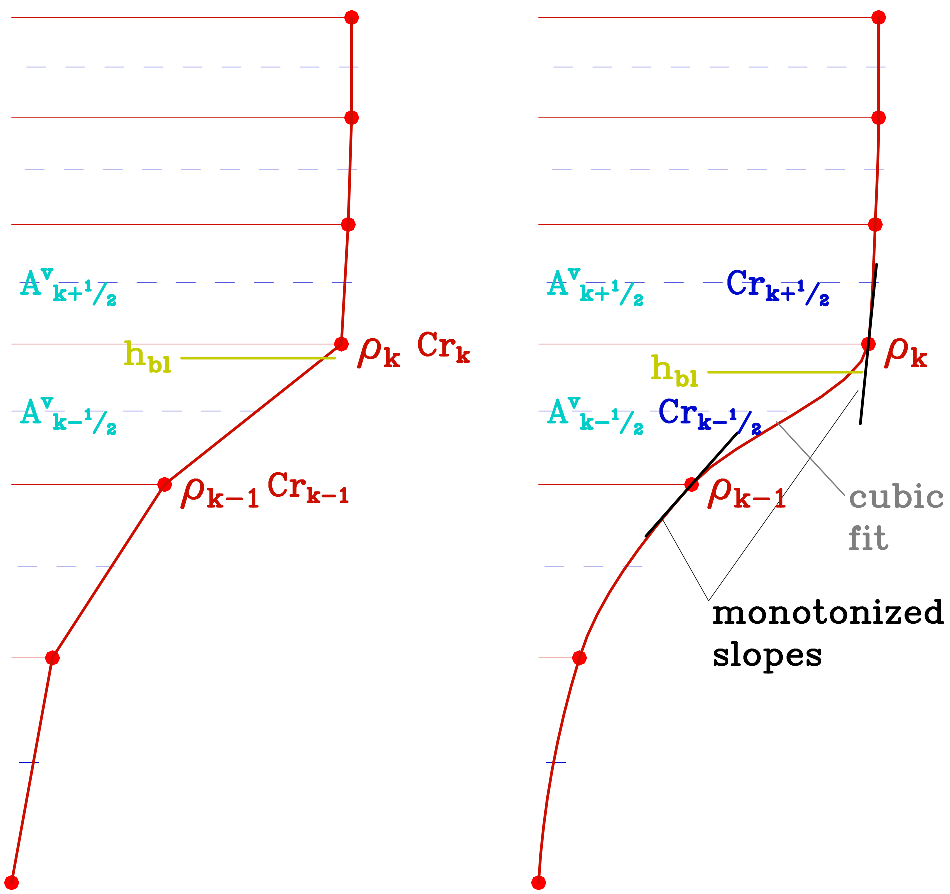
- Estimate V_t^2 and $Cr(z)$ at midpoints $z_{k+1/2}$ using monotonized cubic pseudo-spline fit for density to estimate density and its vertical derivate

- *harmonic* averaging of *adiabatic* differences of density field (the same(!) idea as for computing horizontal pressure gradient)

$\Rightarrow$ unlocking vertical steppiness

$\Rightarrow$ typically shallower BL

less steppiness $\Rightarrow$ more horizontal noise in h_{bl} in 3D model



Cubic fit is to compute

$$V_t^2{}_{k+1/2} \qquad Cr_{k+1/2}$$

not to interpolate it to find h_{bl}

Due to

$$Cr \sim w_* \sqrt{N^2 - N^2 d}$$

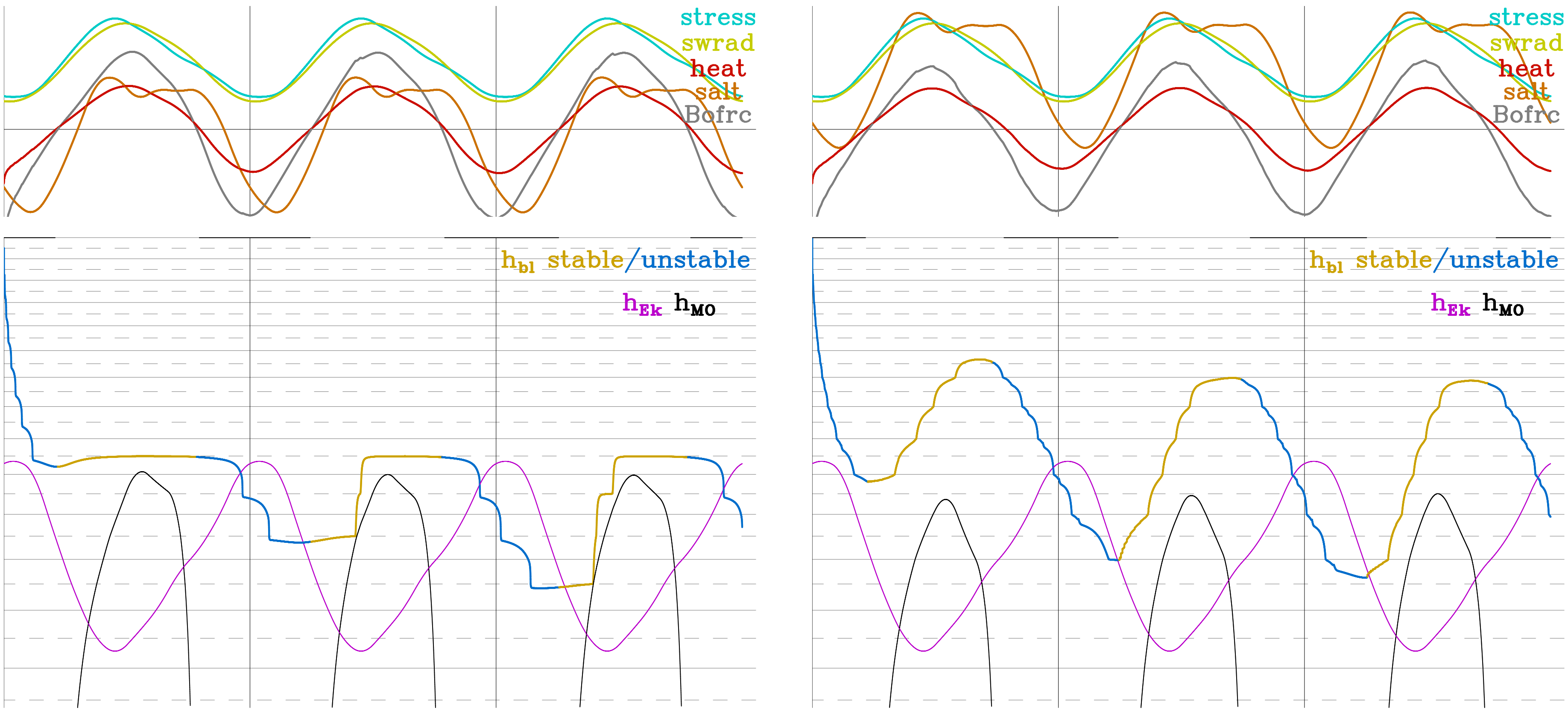
$Cr(z)$ is **not monotonic** near

$$z = -h_{\mathsf{bl}}$$

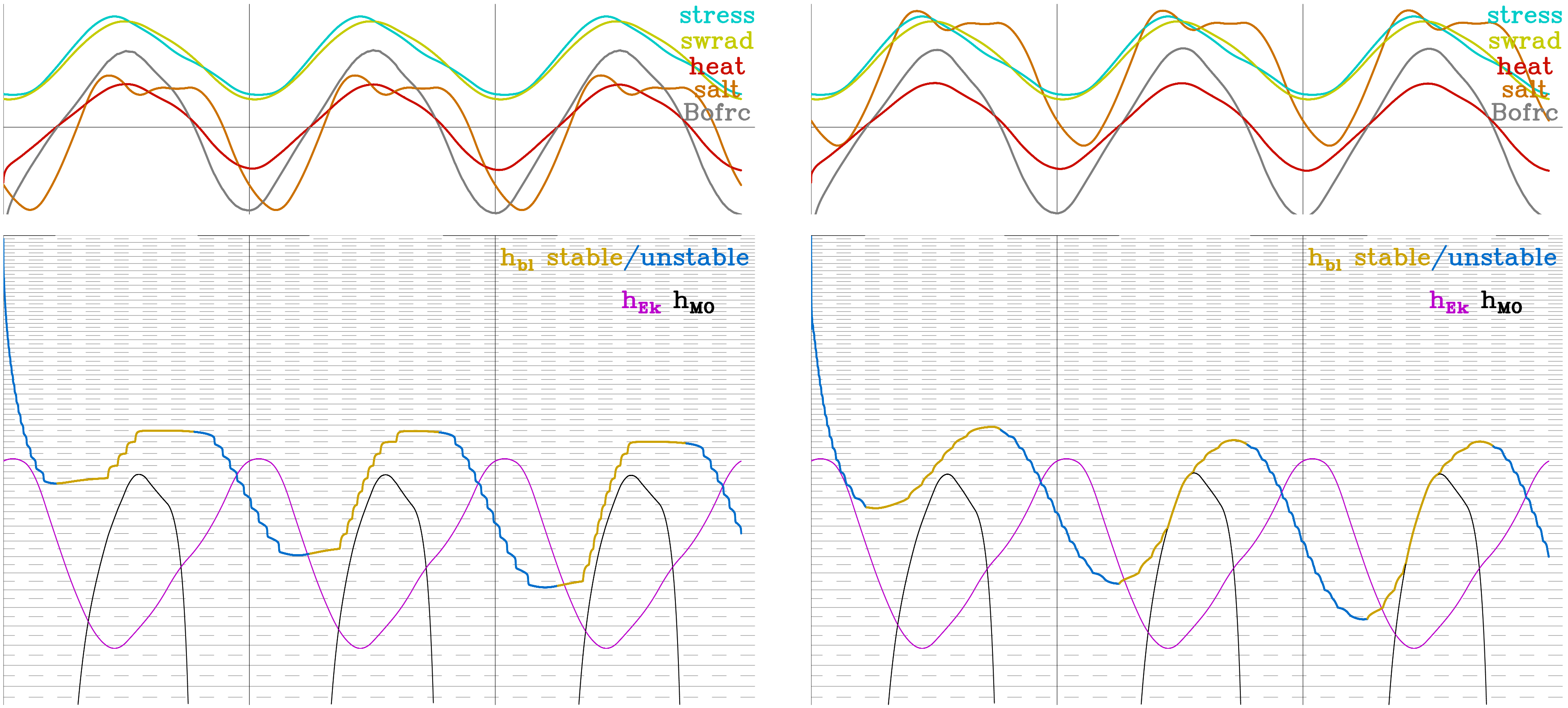
even if $\rho(z)$ and $u, v(z)$ are

$\Rightarrow$ quadratic (cubic) interpolation of Ri_b or Cr is dangerous

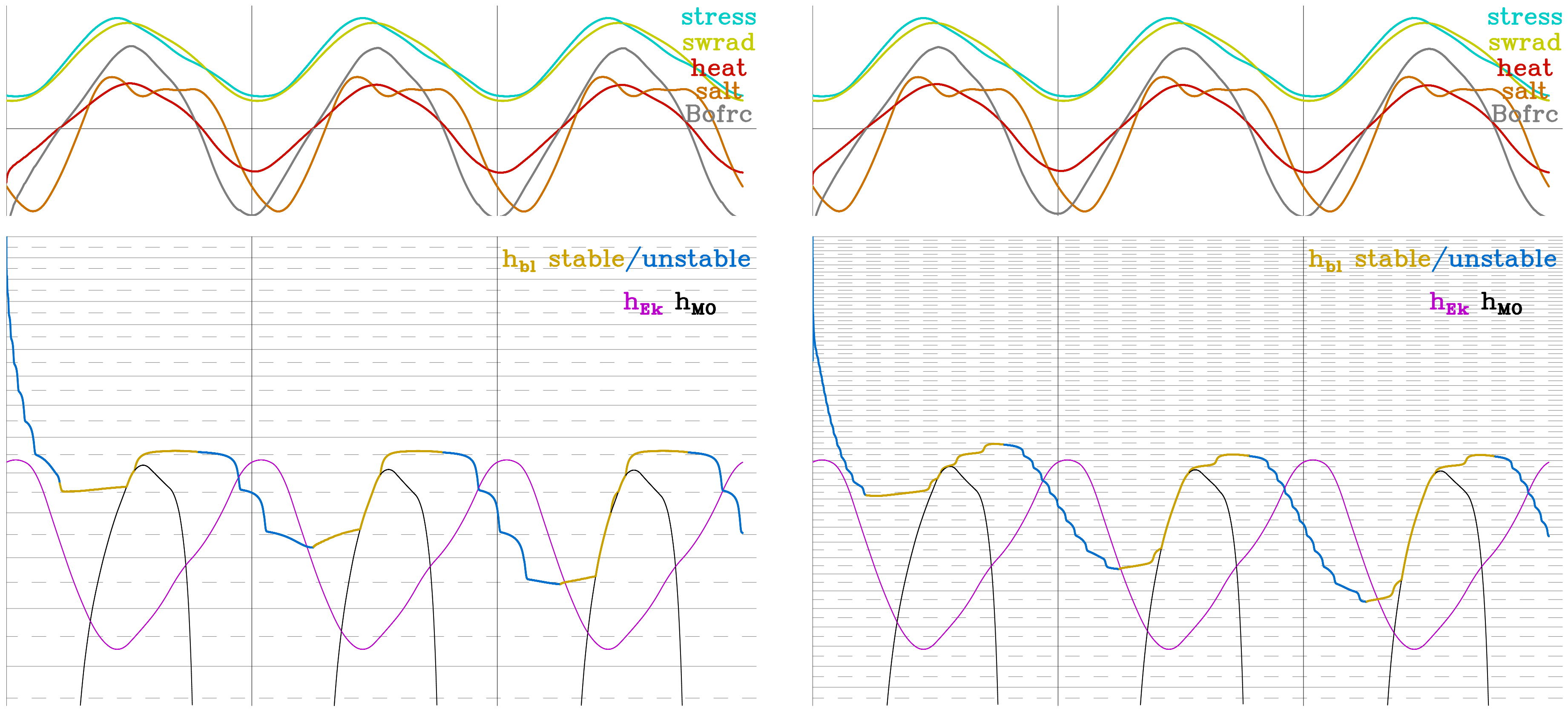
Overall the major source of numerical sensitivity



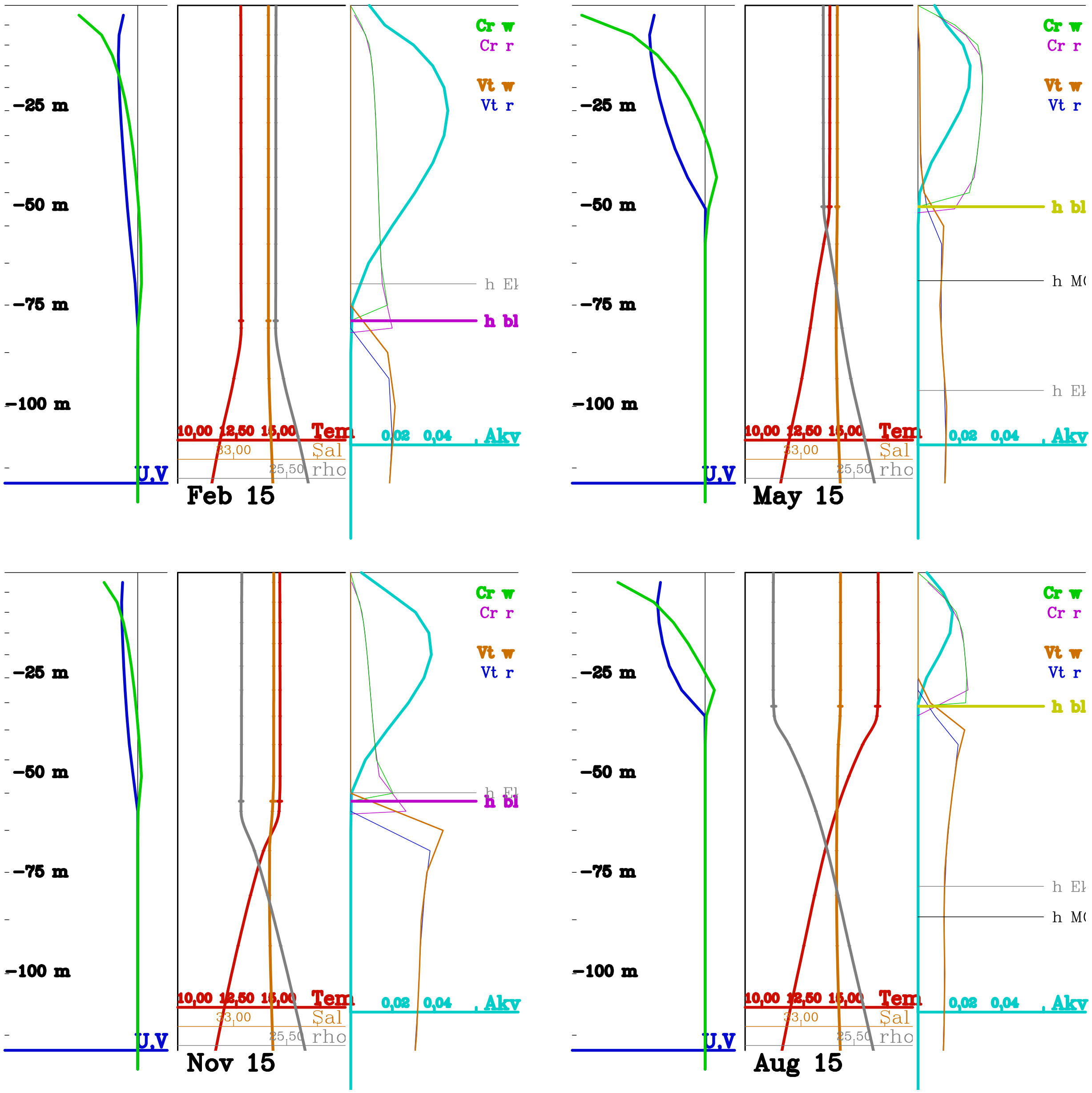
Cr at ρ vs. W-points: **unlocking vertical steps**, N=40



Cr at ρ vs. W-points: N=120



$Cr(z)$ at midpoints $z_{k+1/2}$, but using cubic interpolation for $Cr(z)$ to find h_{bl}: PBL is deeper (consistent with W. Large and G. Danabasoglu), less difference between high and low resolution, but **steppiness is back**.


-25 m
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Cr w
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Feb 15
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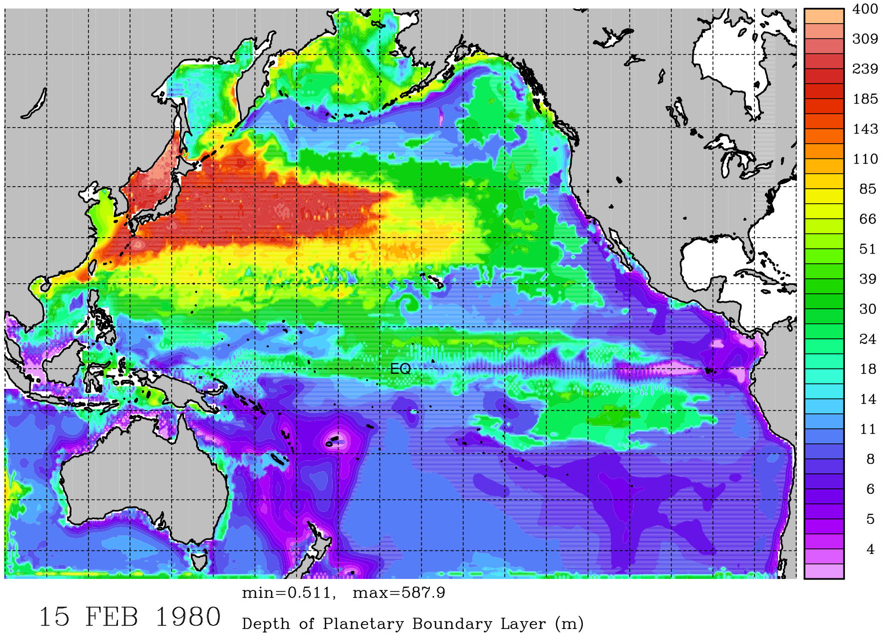
This is why.

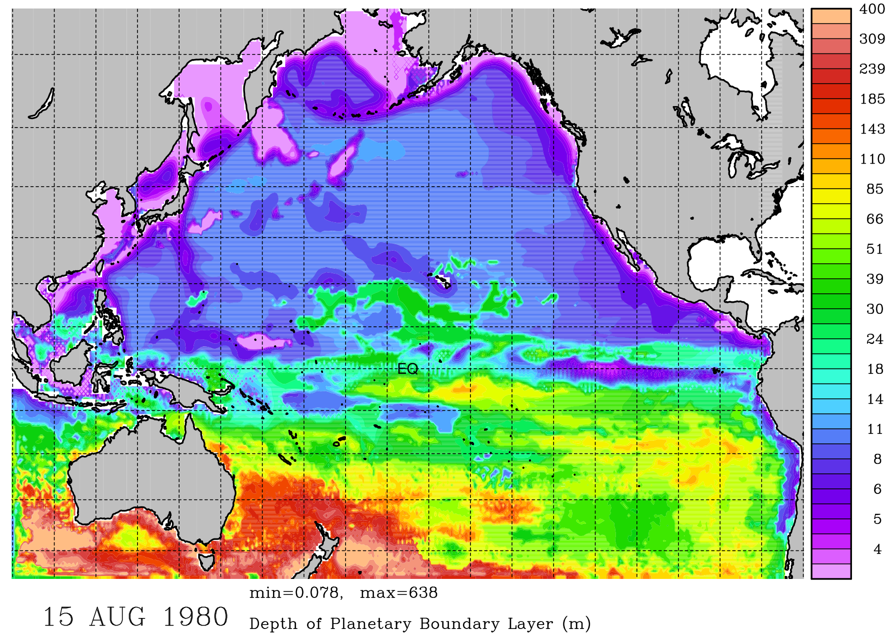
3D Modeling

1/2 degree ROMS Pacific model

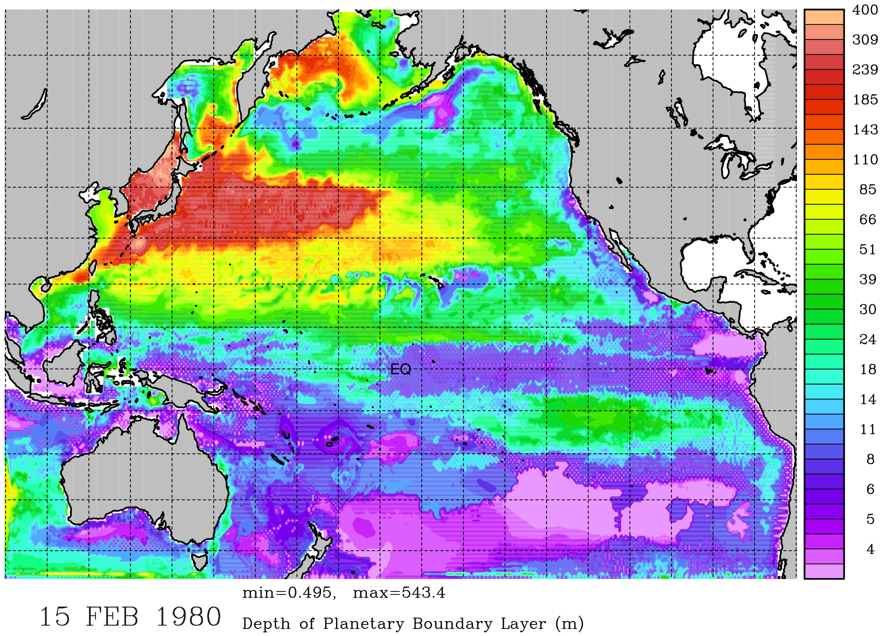
$384 \times 224 \times 32$ grid

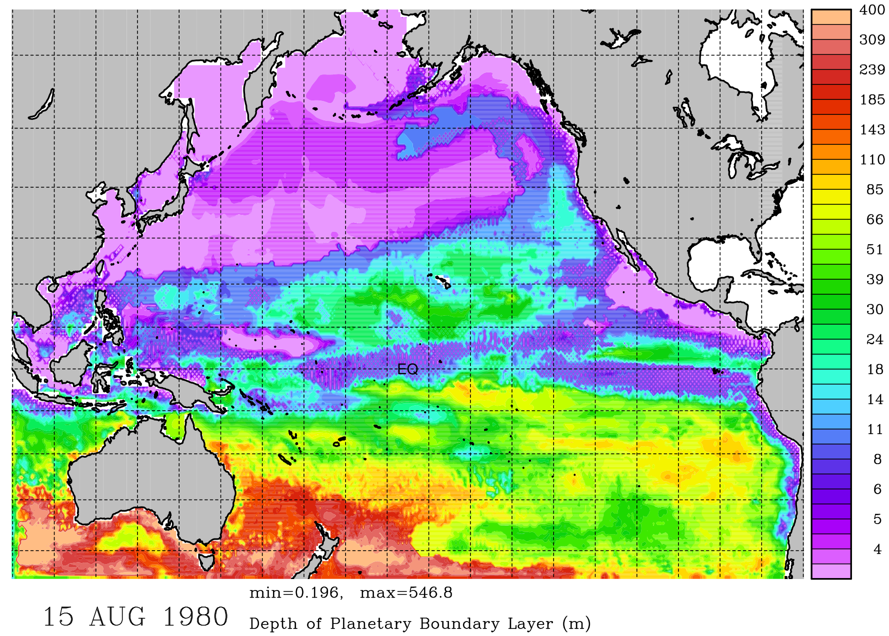
driven by NCEP winds



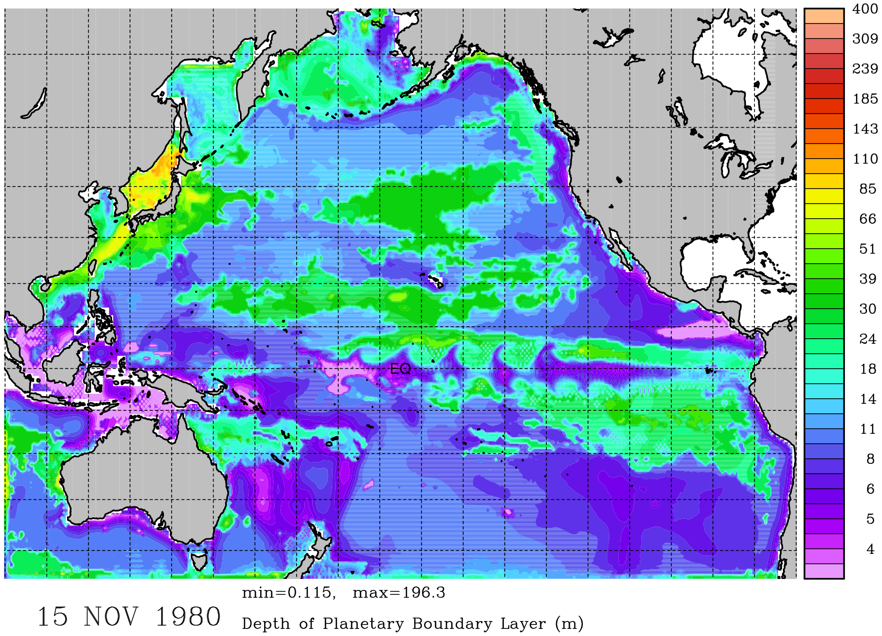


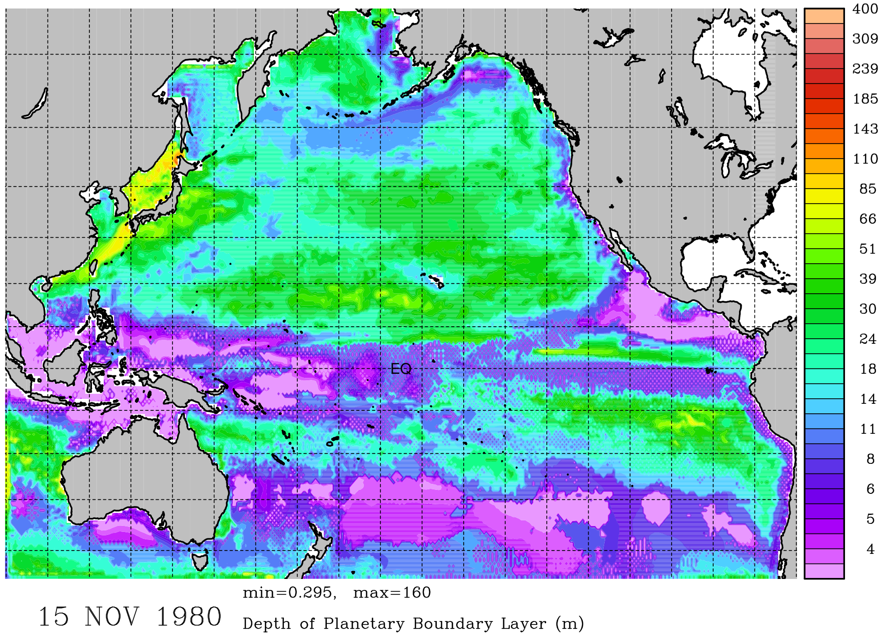
Winter/summer HBL, **20 layers**, basic (1998) KPP numerics
no smoothing of any kind



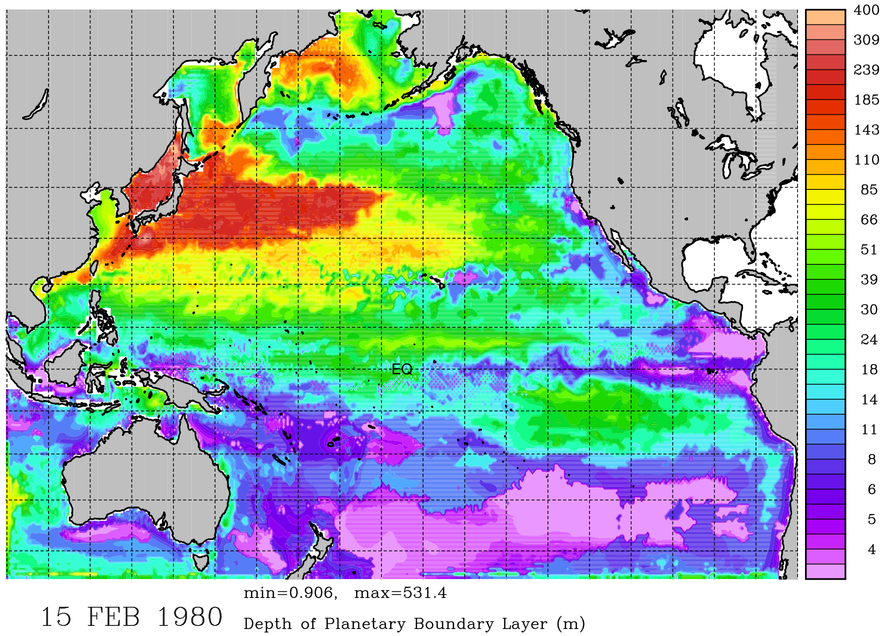


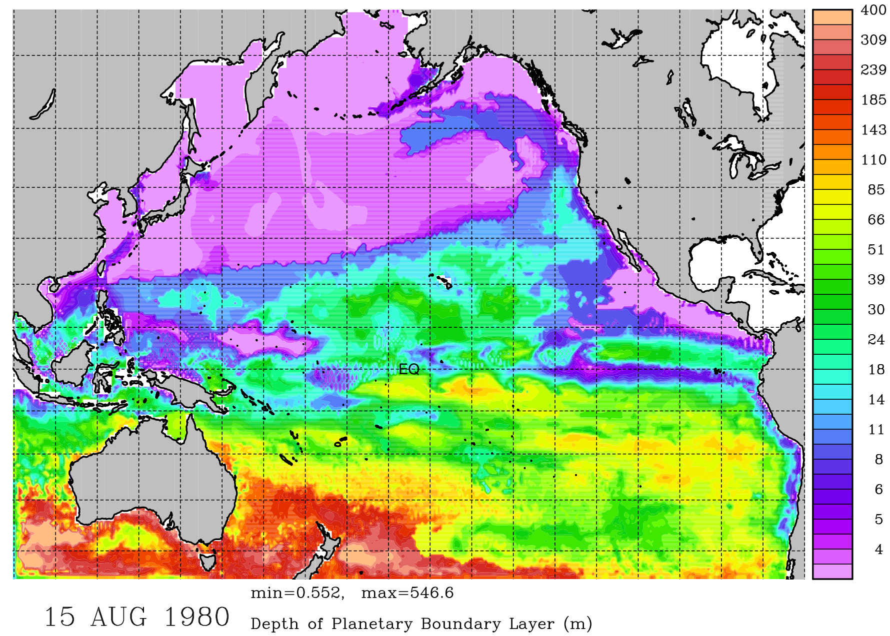
Winter/summer HBL, **30 layers**, basic KPP numerics, **no smoothing of any kind**



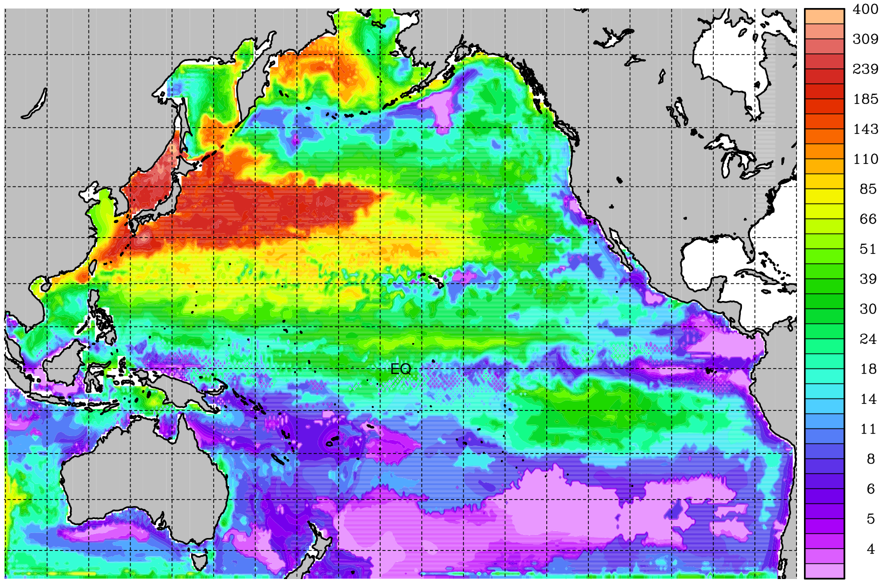


20 vs. 30 layer results, basic (1998) KPP numerics, **no smoothing of any kind**



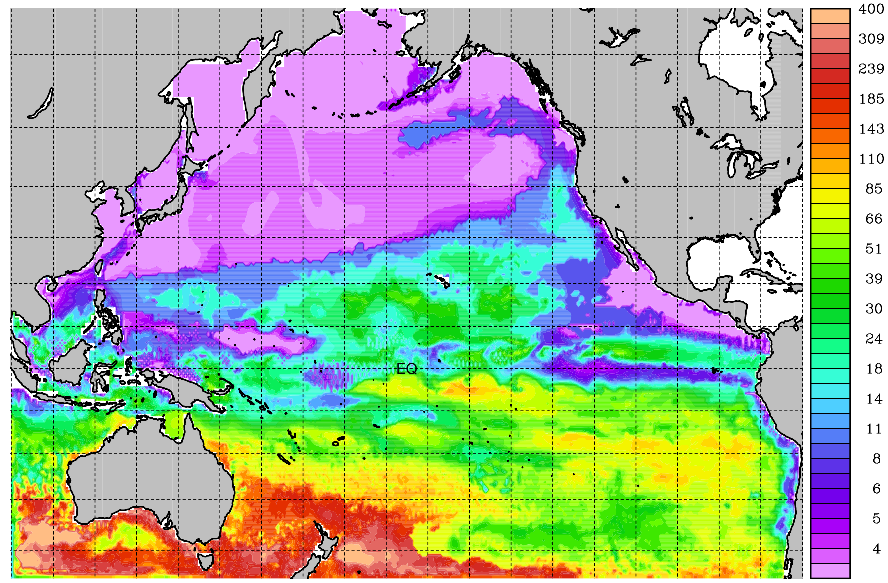


bulk difference Ri_b; enhanced baseline code (convec $\rightarrow$ end and below HBL only; 2-point matching for Ak_v, Ak_t in/out HBL; but ζ is w_* is still limited for unstable B_f only, (Large, 1994; NCOM 1998)



min=0.925, max=531.4

15 FEB 1980 Depth of Planetary Boundary Layer (m)

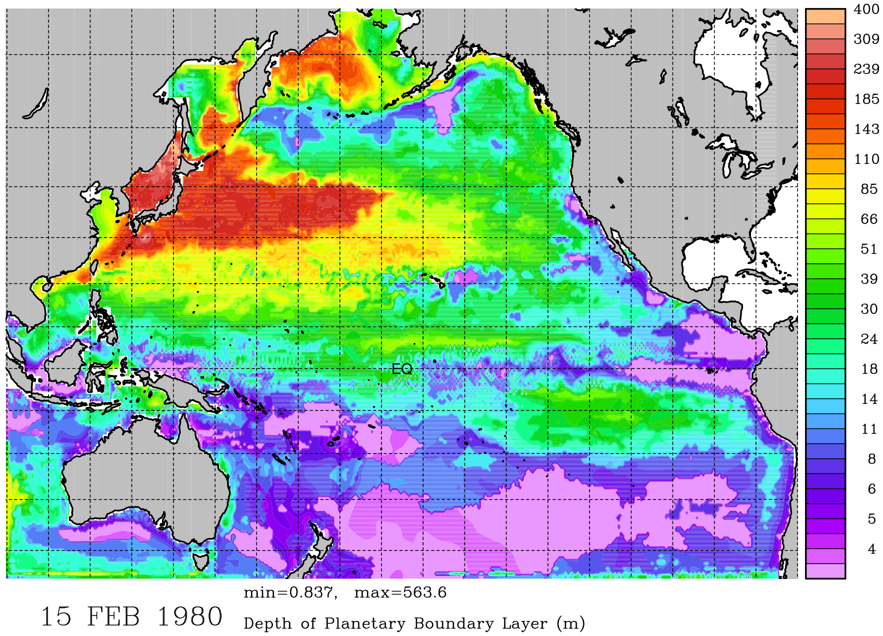


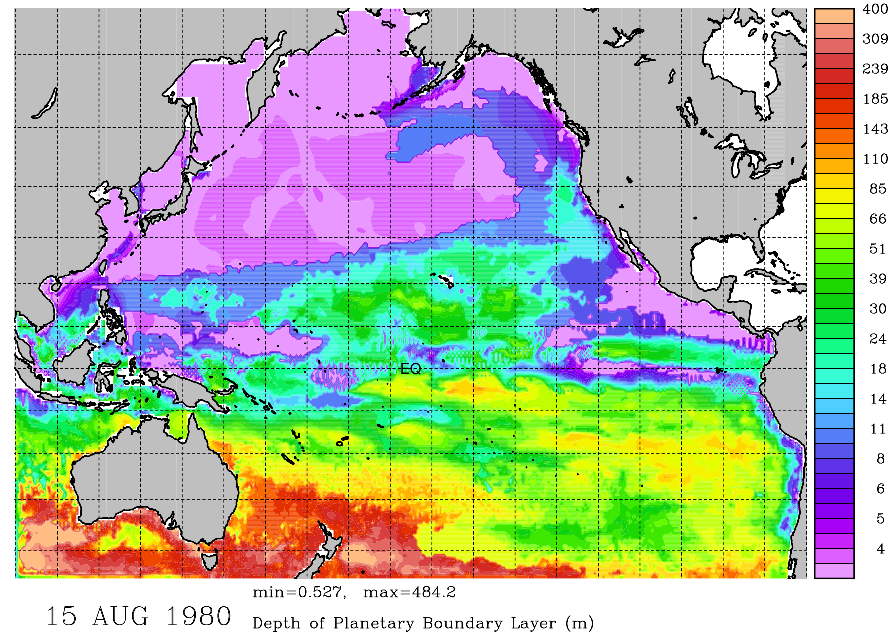
min=0.554, max=546.6

15 AUG 1980 Depth of Planetary Boundary Layer (m)

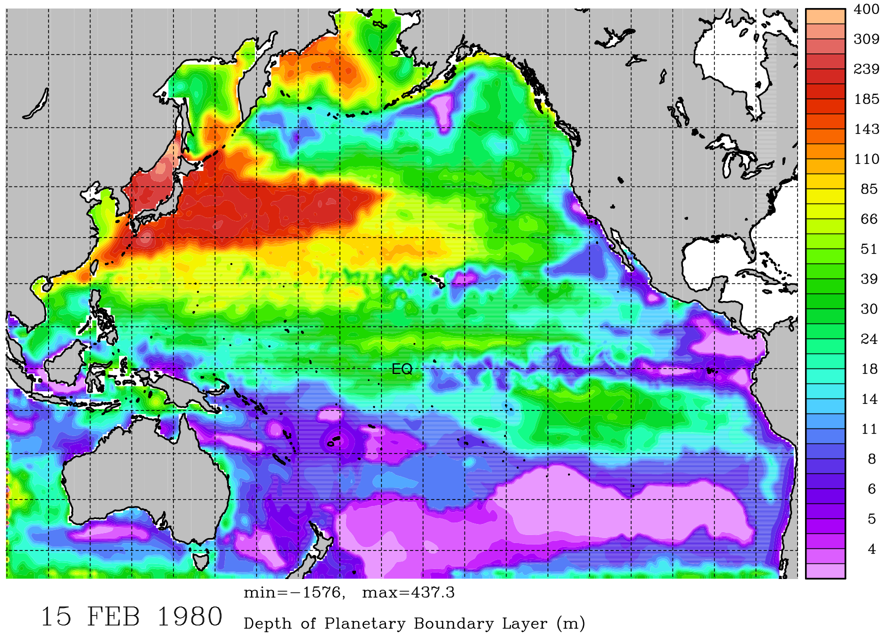
bulk difference Ri_b, symmetric ζ-limiting ($\approx$ NCAR 2003)

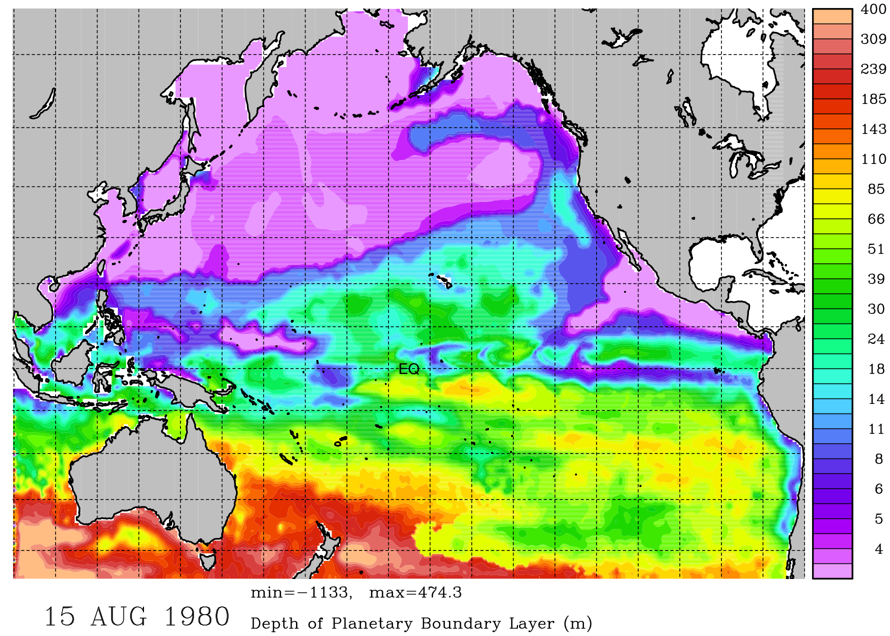
this is new baseline code



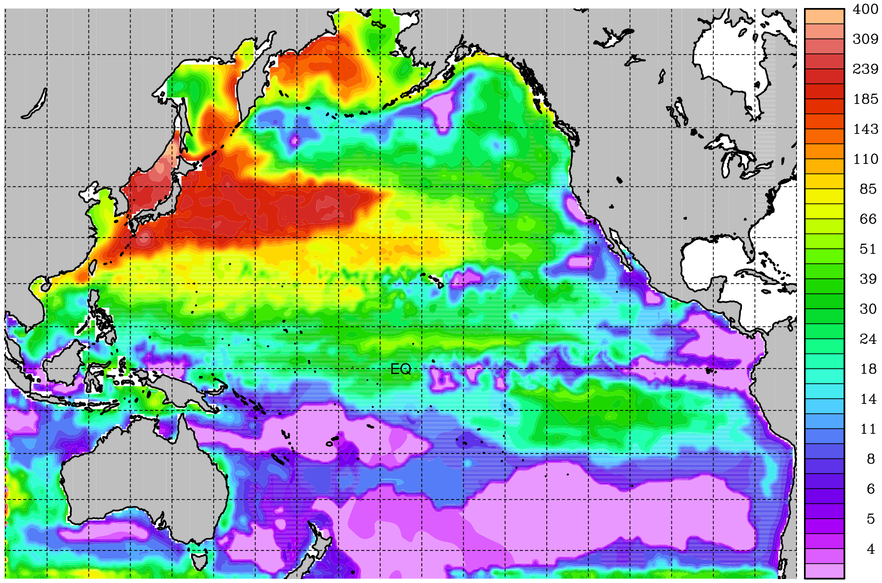


integral Ri_b, all other features like is new baseline



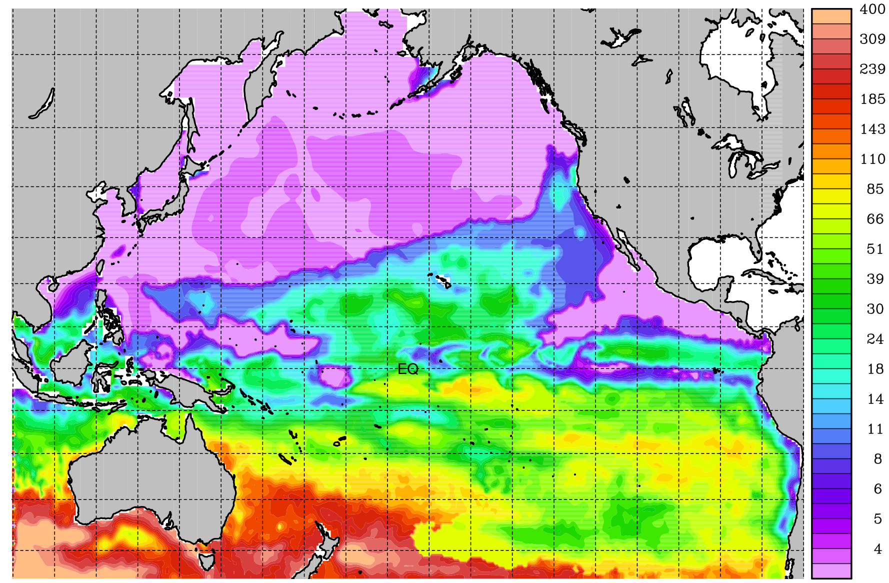


bulk difference Ri_b, horizontally averaged HBL



min=−1557, max=452.4

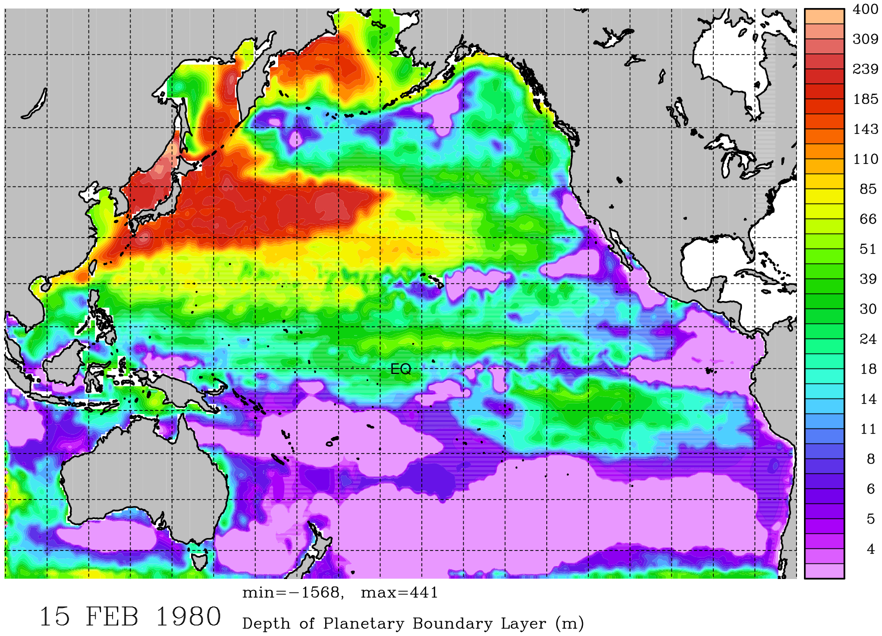
15 FEB 1980 Depth of Planetary Boundary Layer (m)

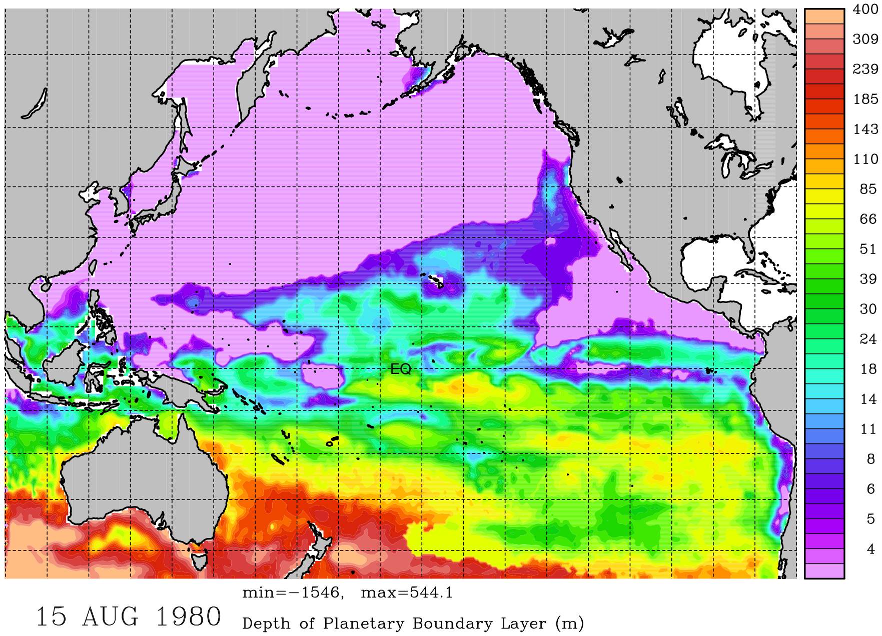


min=−966.9, max=523.1

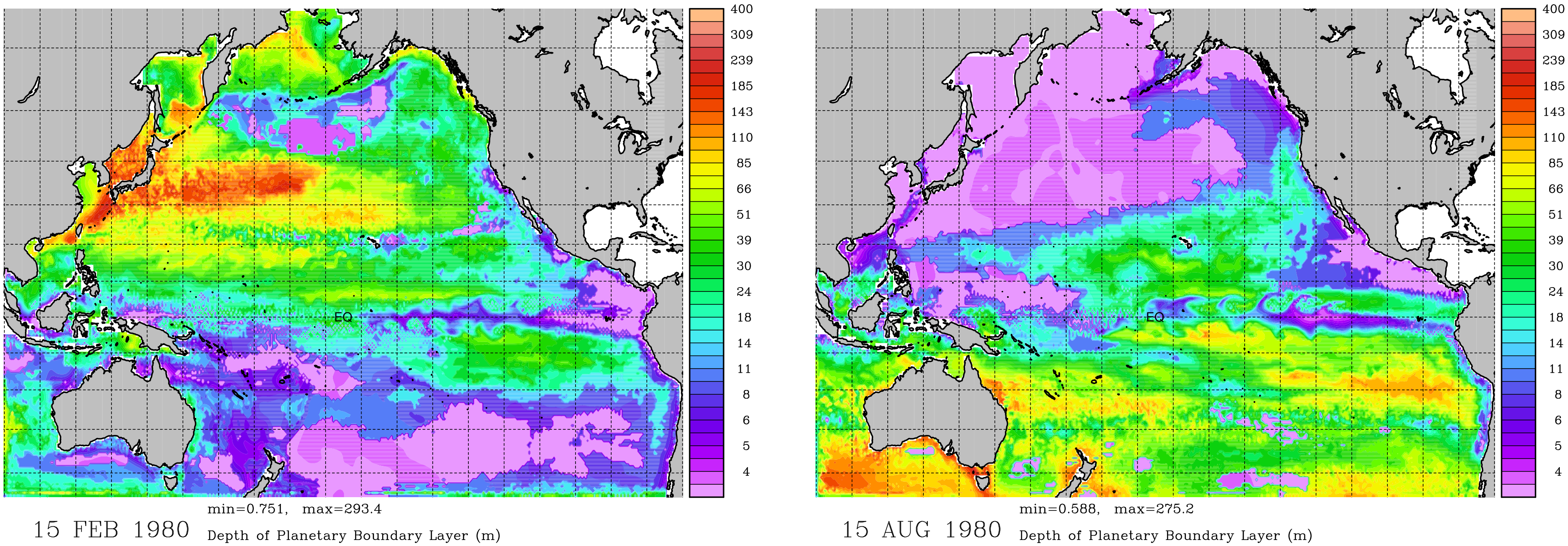
15 AUG 1980 Depth of Planetary Boundary Layer (m)

integral Ri_b at ρ-points, horizontally averaged HBL

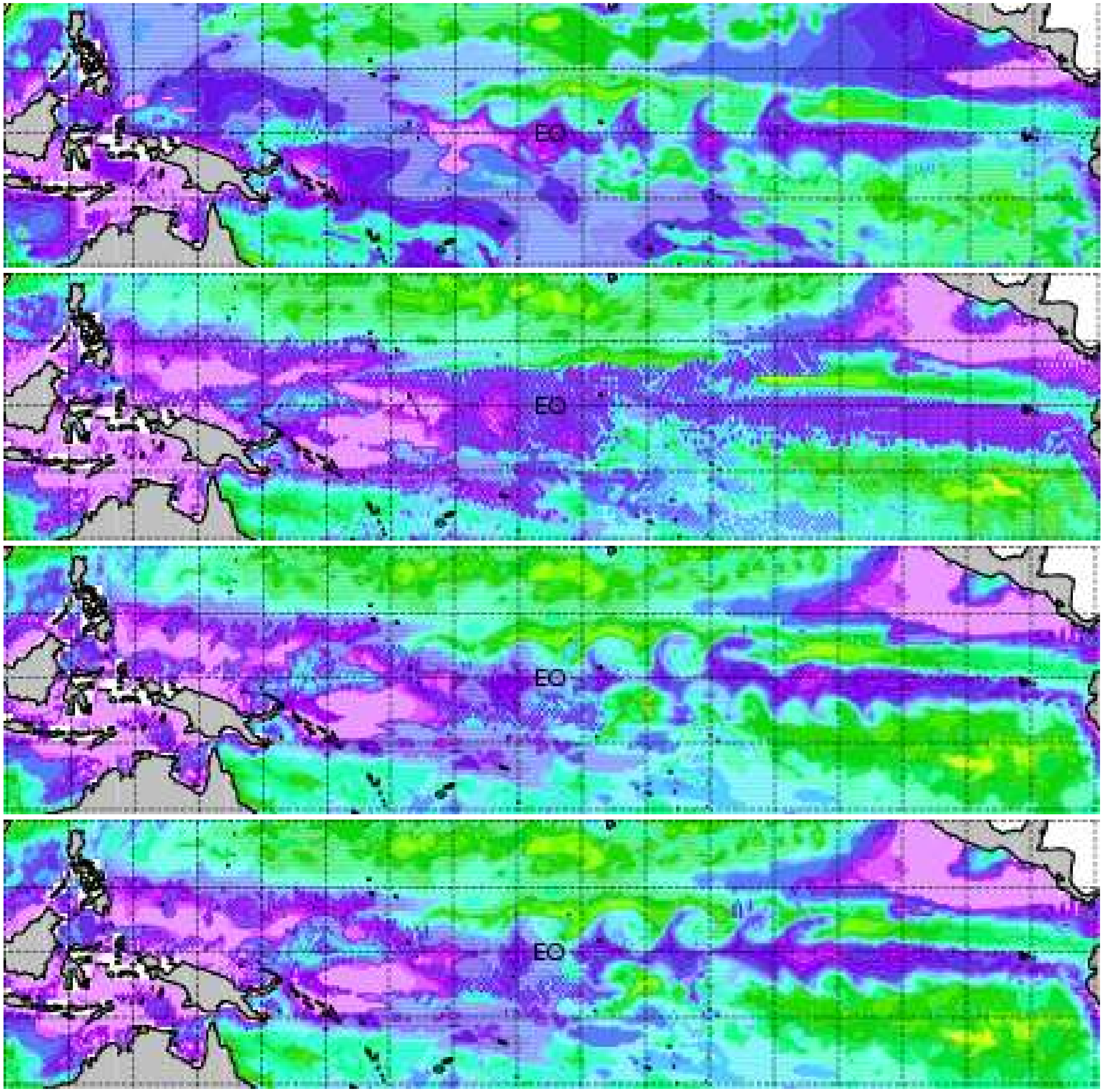




integral Ri_b an W-points, horizontally averaged HBL



integral Ri_b, **new Coriolis**, $Ri_{cr} = 0.45$, no averaging of HBL



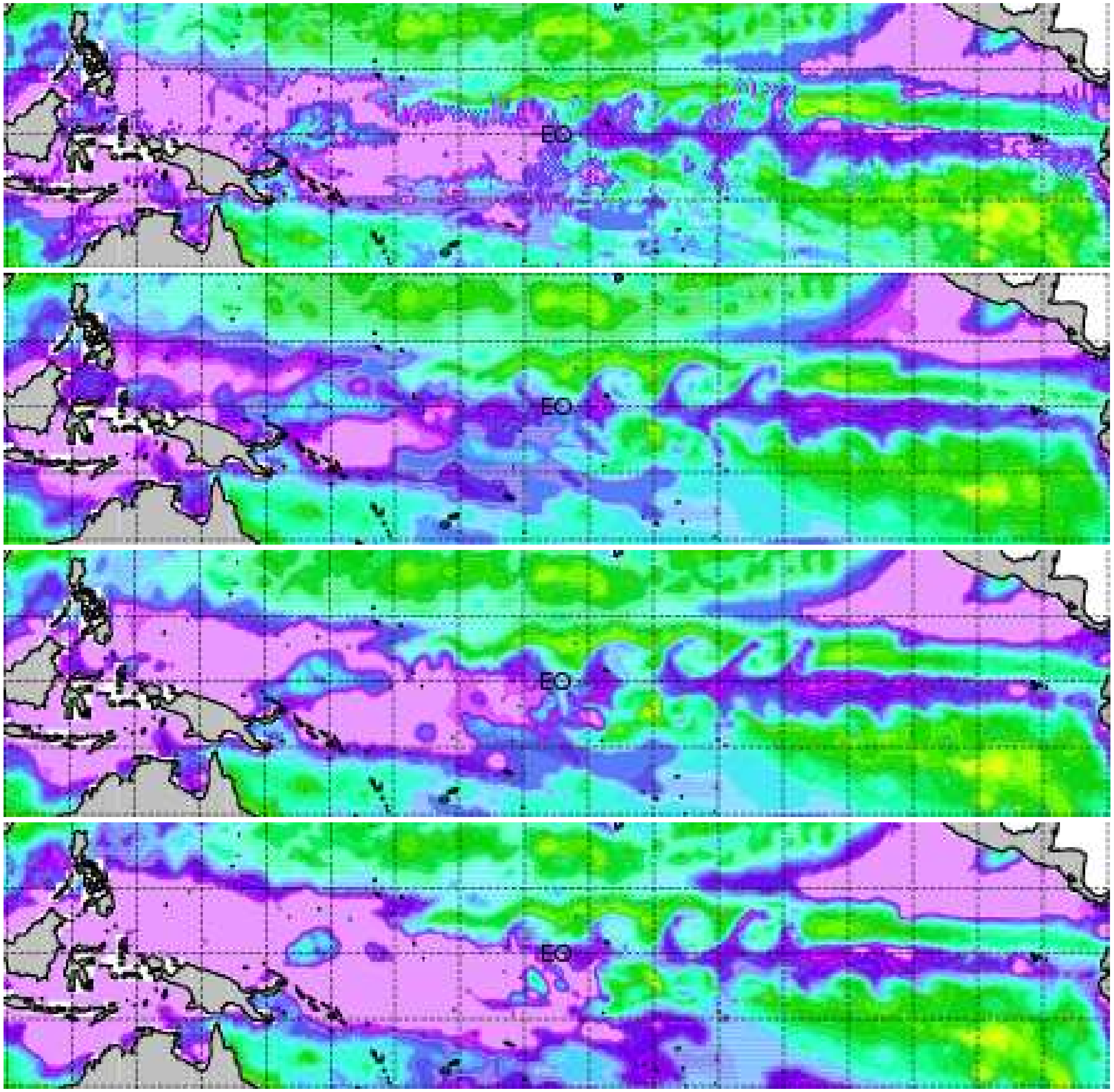
N=20, 1998

N=30, 1998

bulk Ri_b
enhanced
baseline code

bulk Ri_b
new baseline

Autumn HBL in Equator from different versions KPP



integral Ri_b

bulk Ri_b
averaged HBL

int Ri_b at ρ-pts
averaged HBL

int Ri_b at W-pts
averaged HBL

Autumn HBL in Equator from different versions KPP

Summary

- Most (not all) updates from W. Large and G. Danabasoglu, 2003
- Modified KPP for compatibility with free surface of ROMS (fixed blow-ups in shallow regions)
- New Monin-Obukhov depth limit similar, but not the same as Large — Danabasoglu
- New options for Ri_b/Cr definition and numerics
- New treatment of Ekman boundary layer

Unsettled

??? Inline Monin-Obukhov limit into Ri_b/Cr — turbulence self-dissipation? (M-Y has such term)

- code is on the ground, but new calibration is needed
- PBL is still too shallow in summer (stable buoyancy forcing)
- still slow convergence
- Nonlocal flux is discontinuous at HBL
- physics below mixed layer remains untouched