Combining a Model with Observations: Data Assimilation in ROMS

Brian Powell
University of Hawaii
(shameless stealing from: Andy Moore, Bruce Cornuelle, Manu Di Lorenzo, Ralph Milliff)

P.-S. Laplace, 1749—1827

P.-S. Laplace, 1749—1827

Laplace’s Demon

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

P.-S. Laplace, 1749—1827

Laplace's Demon

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

P.-S. Laplace, 1749—1827

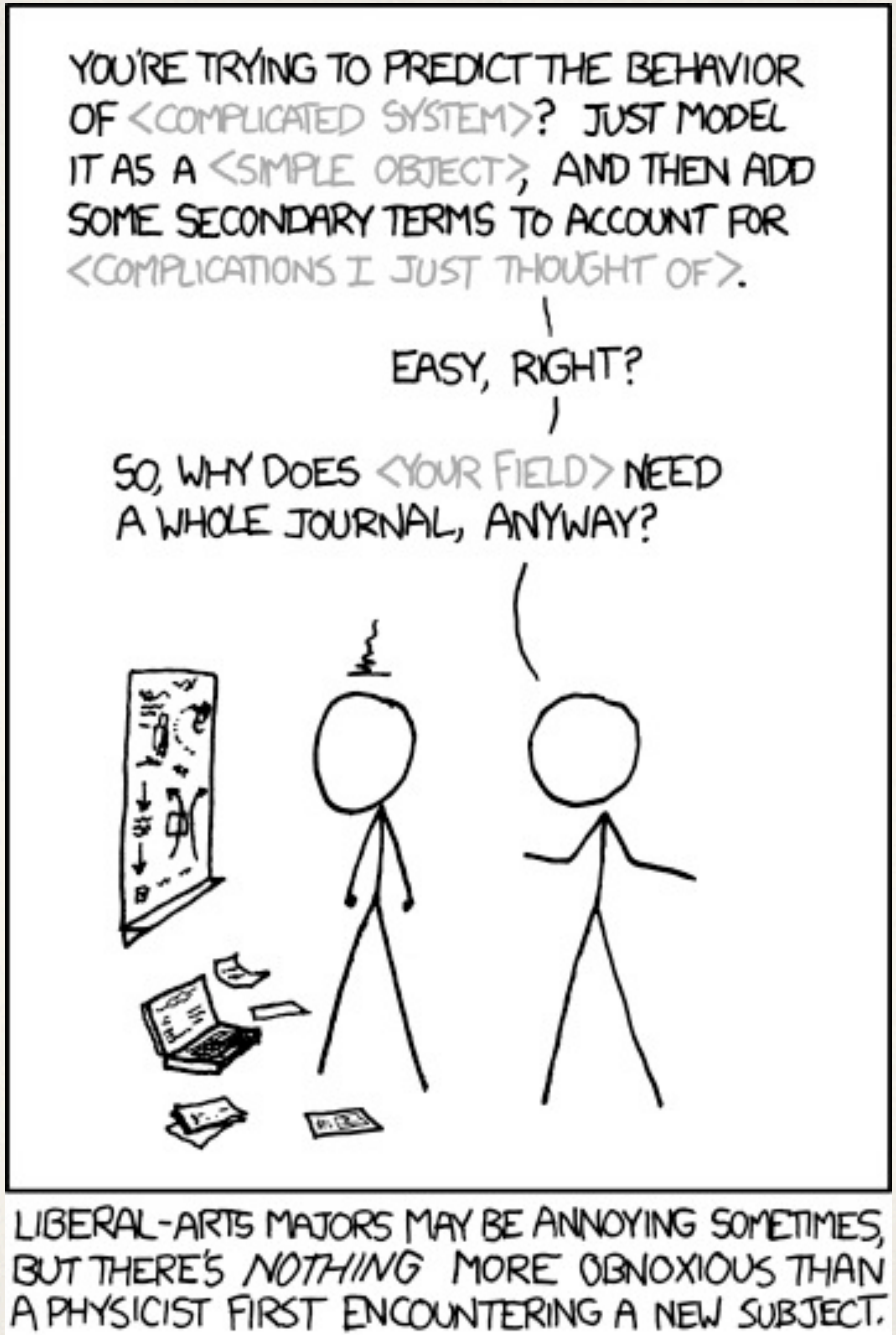
Laplace’s Demon

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.

P.-S. Laplace, 1749—1827

Laplace's Demon

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.



YOU'RE TRYING TO PREDICT THE BEHAVIOR OF <COMPLICATED SYSTEM>? JUST MODEL IT AS A <SIMPLE OBJECT>, AND THEN ADD SOME SECONDARY TERMS TO ACCOUNT FOR <COMPLICATIONS I JUST THOUGHT OF>.
EASY, RIGHT?
SO, WHY DOES <YOUR FIELD> NEED A WHOLE JOURNAL, ANYWAY?
LIBERAL-ARTS MAJORS MAY BE ANNOYING SOMETIMES, BUT THERE'S *NOTHING* MORE OBNOXIOUS THAN A PHYSICIST FIRST ENCOUNTERING A NEW SUBJECT.

“All forces that set nature in motion”

$$\frac{\partial u}{\partial t} + \vec{v} \cdot \nabla u - f v = -\frac{\partial \phi}{\partial x} - \frac{\partial}{\partial z}\left(\overline{u'w'} - \nu \frac{\partial u}{\partial z}\right) + \mathcal{F}_u + \mathcal{D}_u$$

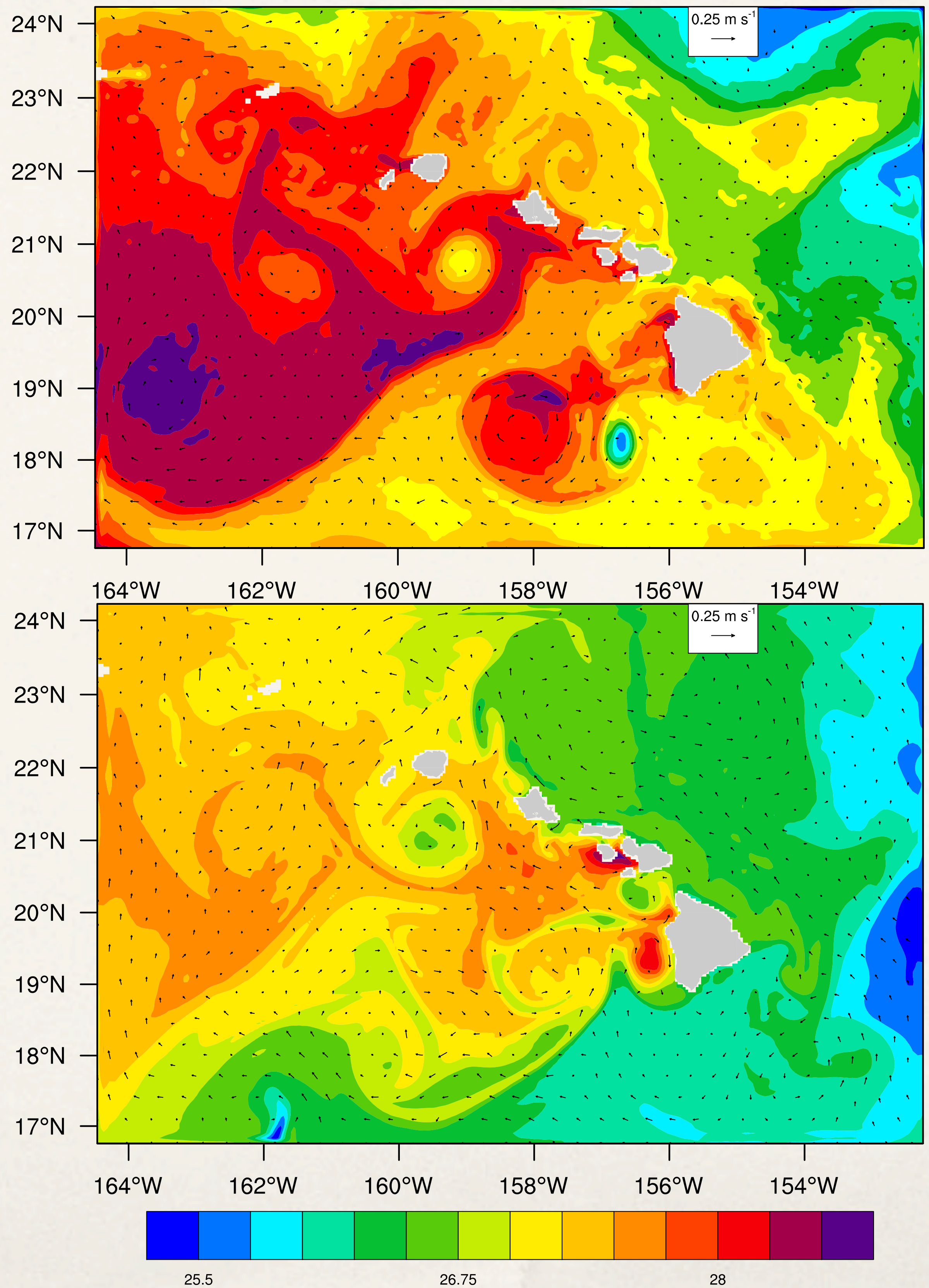
$$\frac{\partial v}{\partial t} + \vec{v} \cdot \nabla v + f u = -\frac{\partial \phi}{\partial y} - \frac{\partial}{\partial z}\left(\overline{v'w'} - \nu \frac{\partial v}{\partial z}\right) + \mathcal{F}_v + \mathcal{D}_v$$

$$\frac{\partial C}{\partial t} + \vec{v} \cdot \nabla C = -\frac{\partial}{\partial z}\left(\overline{C'w'} - \nu_\theta \frac{\partial C}{\partial z}\right) + \mathcal{F}_C + \mathcal{D}_C$$

$$\frac{\partial \phi}{\partial z} = -\frac{\rho g}{\rho_o}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

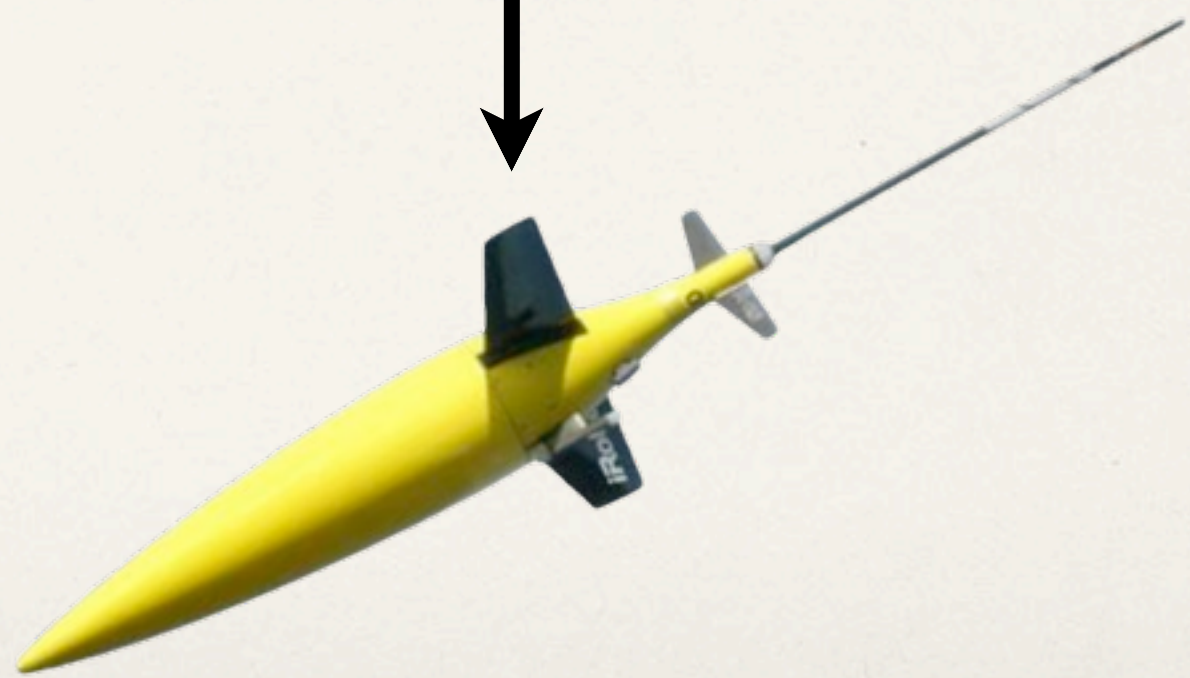
“All positions of all items”



$x \leftarrow$

$\mathcal{H}$

$\downarrow$

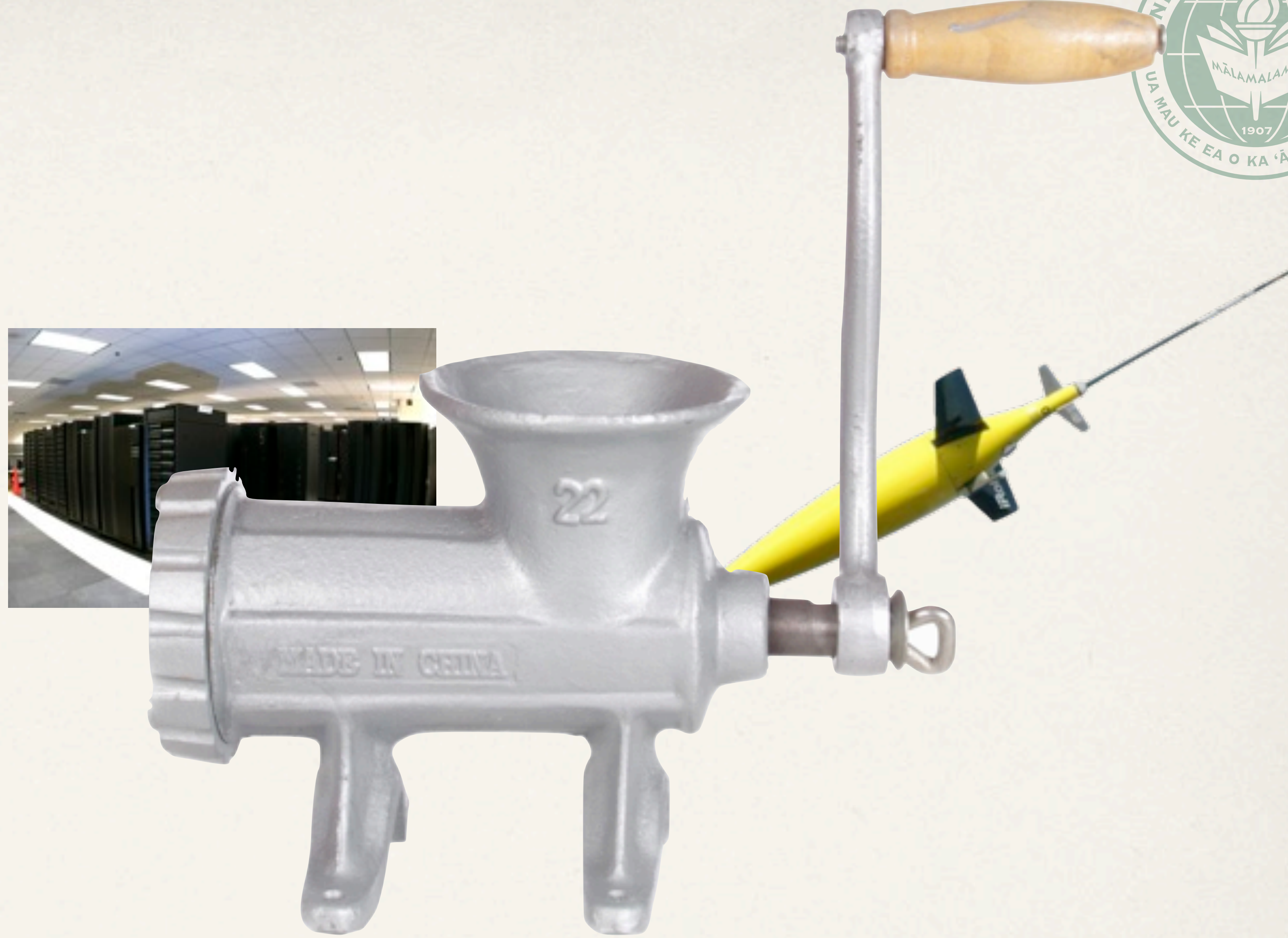


$y \leftarrow$

=

+

=



$$x_a = x + \alpha\left(y - \mathcal{H}x\right)$$

How to determine α ?

- Immediately Replace:

$$\alpha = 1$$

- Over Time:

$$\alpha = \frac{\Delta t}{\tau}$$

How to determine α ?

- Imme
- Over T

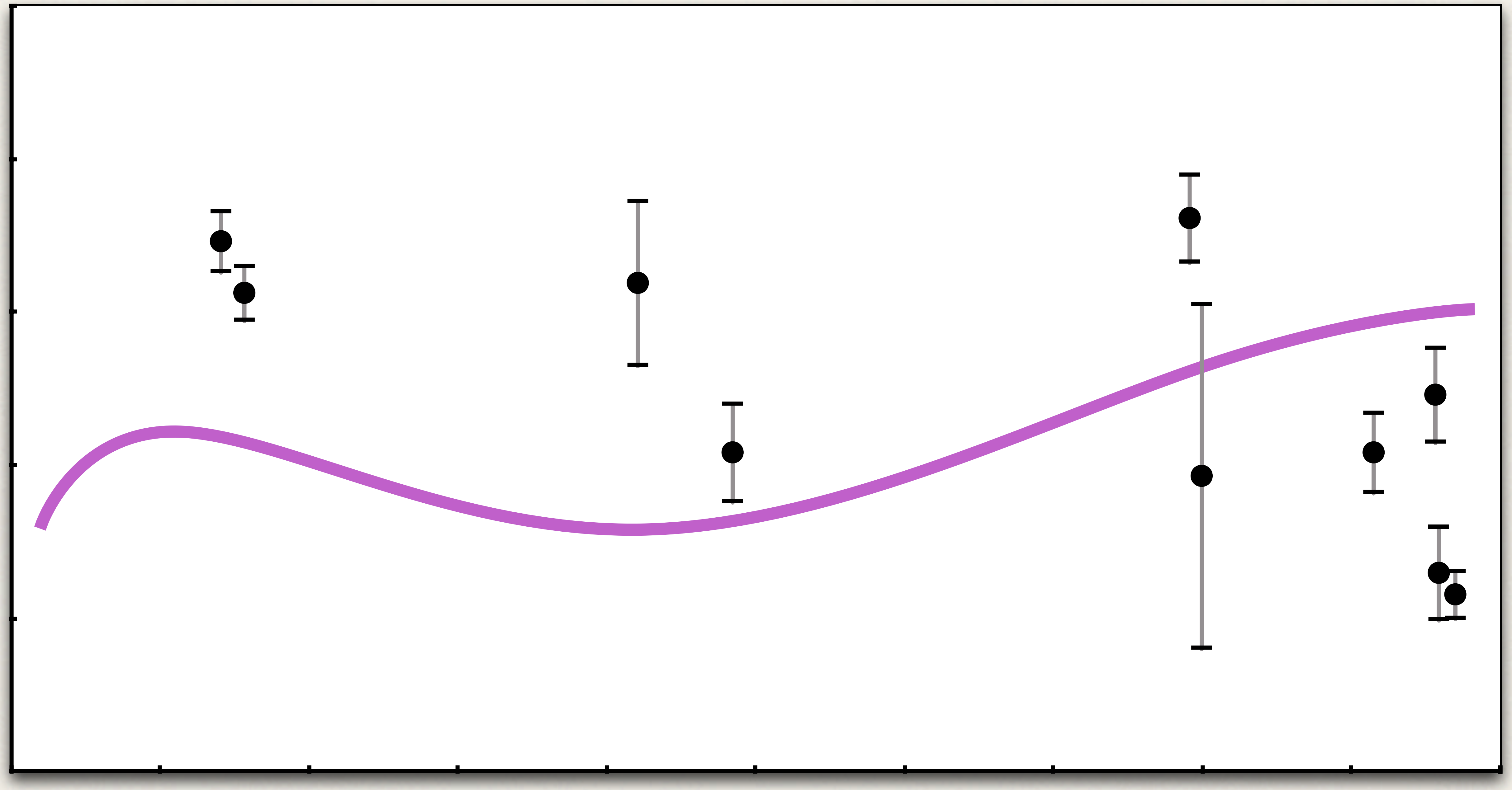
How to determine α ?

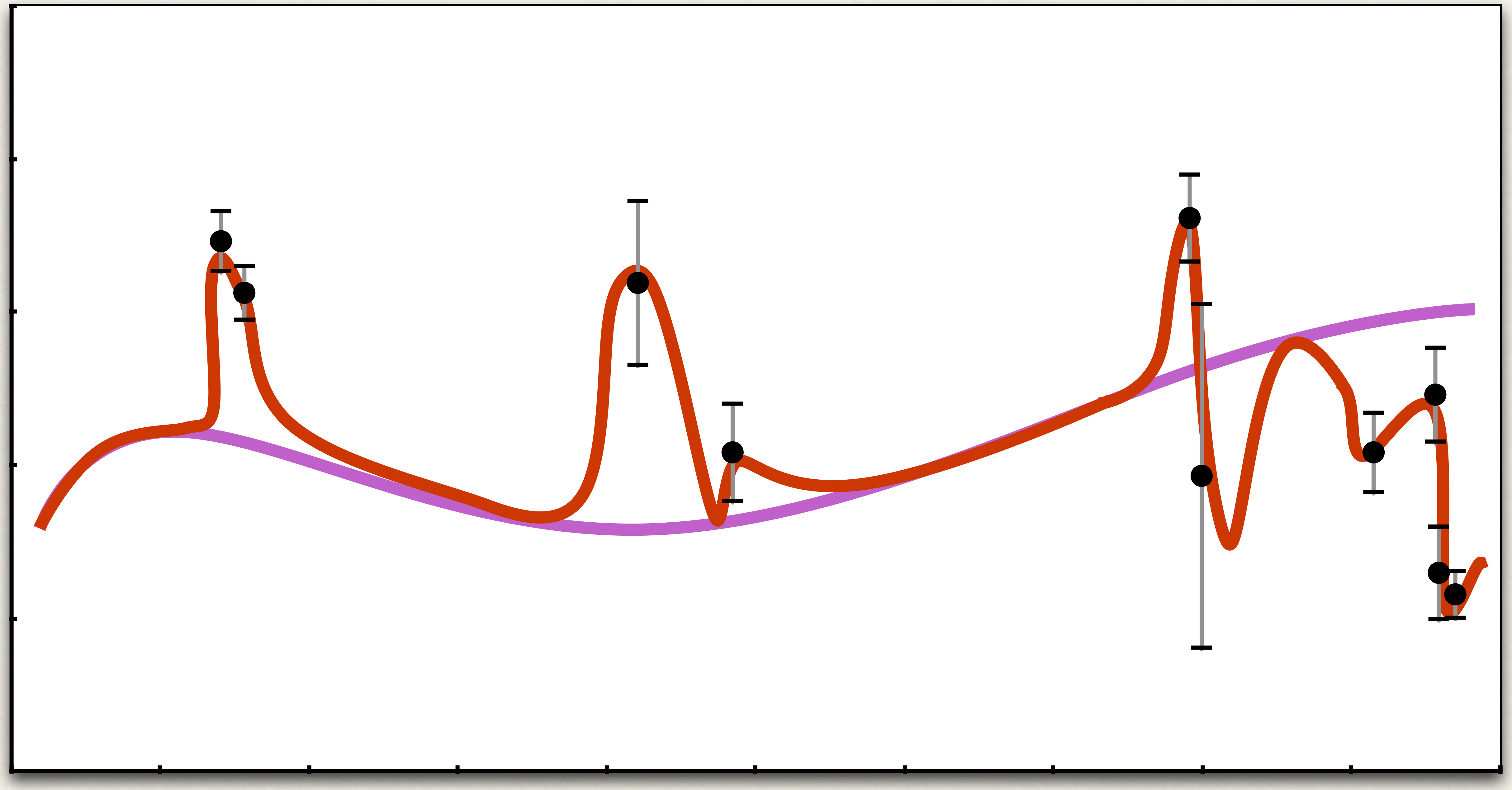
- Immediately Replace:

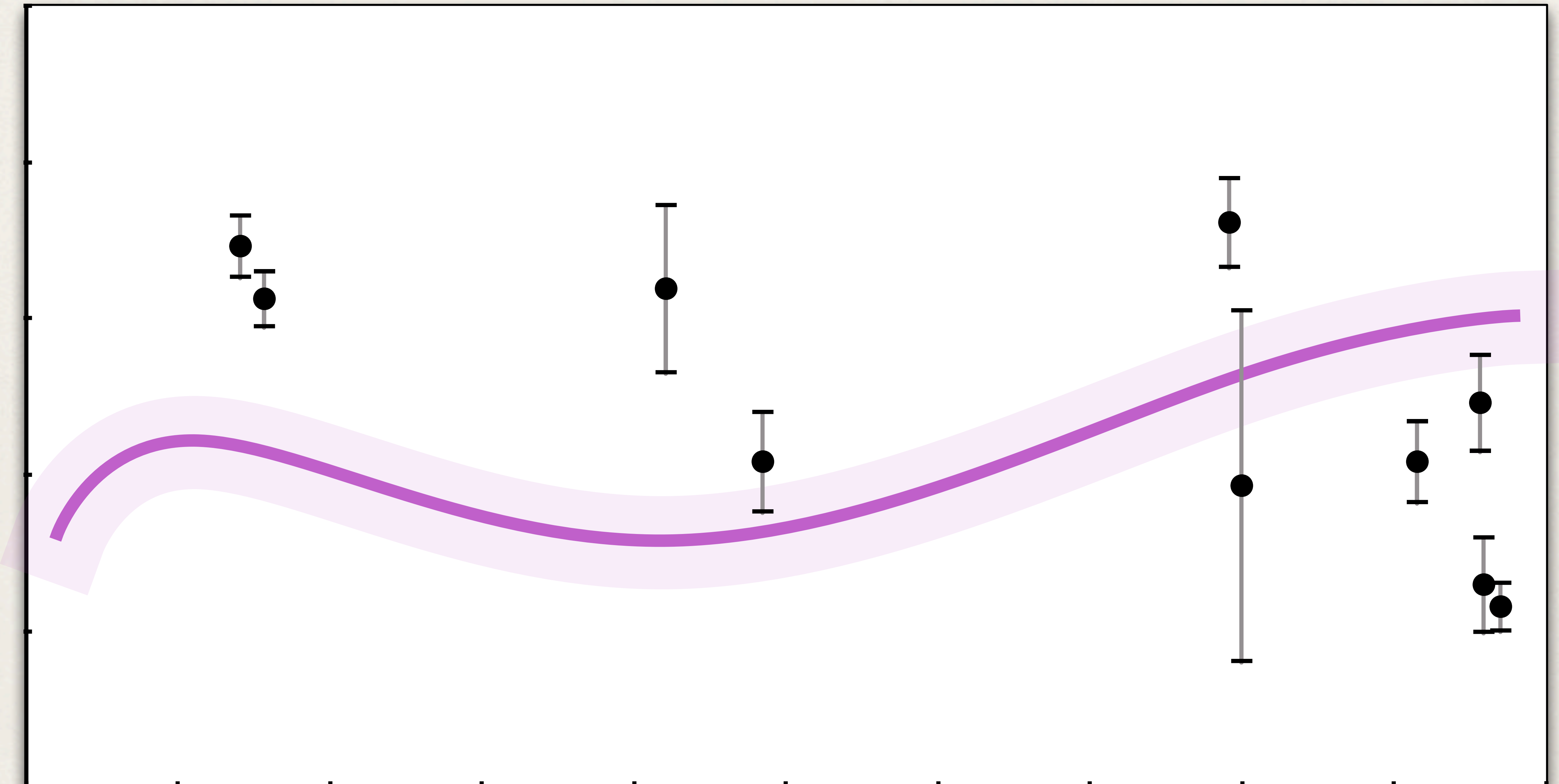
$$\cancel{\alpha = 1}$$

- Over Time:

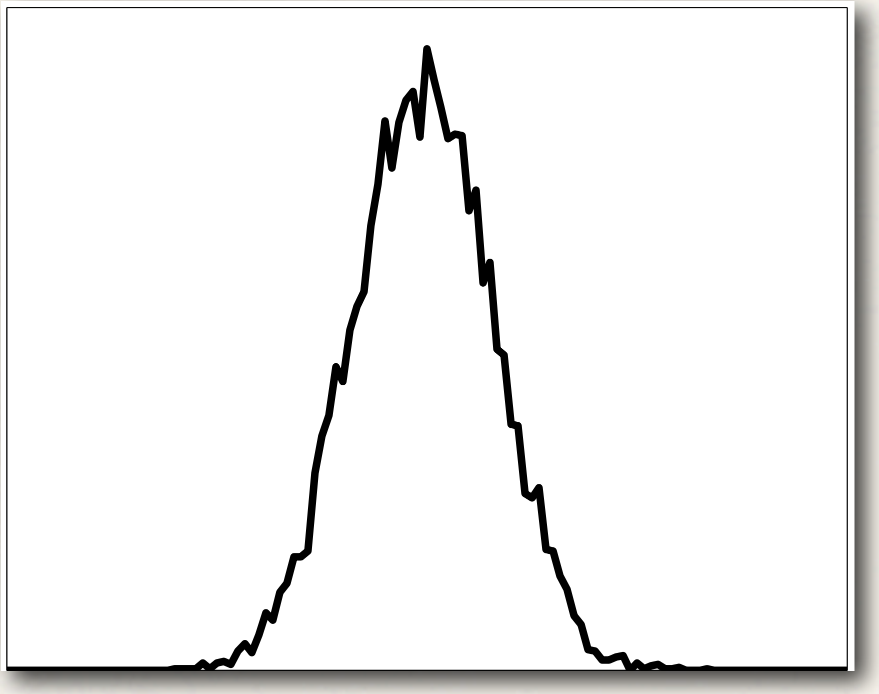
$$\cancel{\alpha = \frac{\Delta t}{\tau}}$$



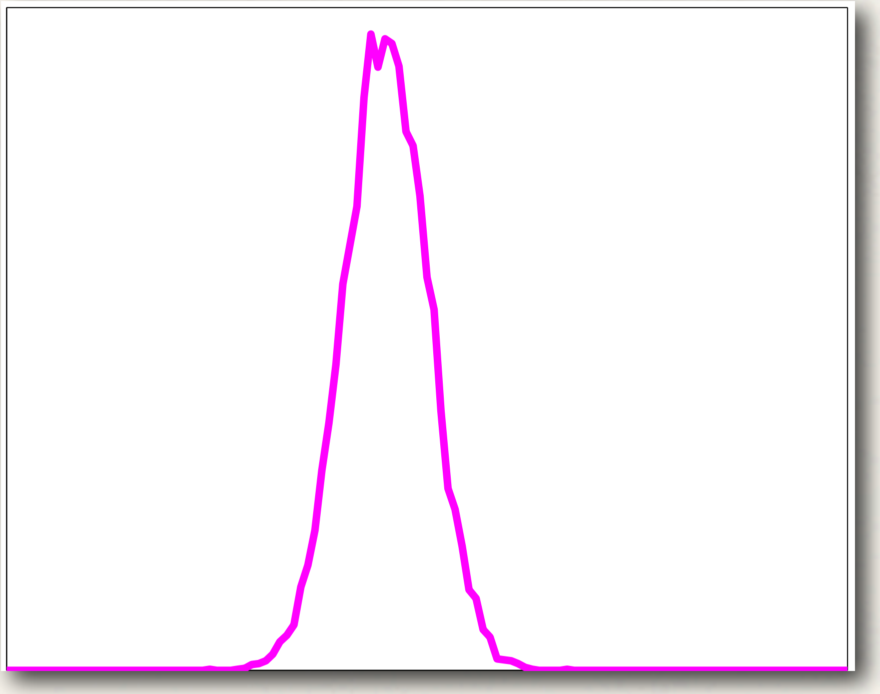




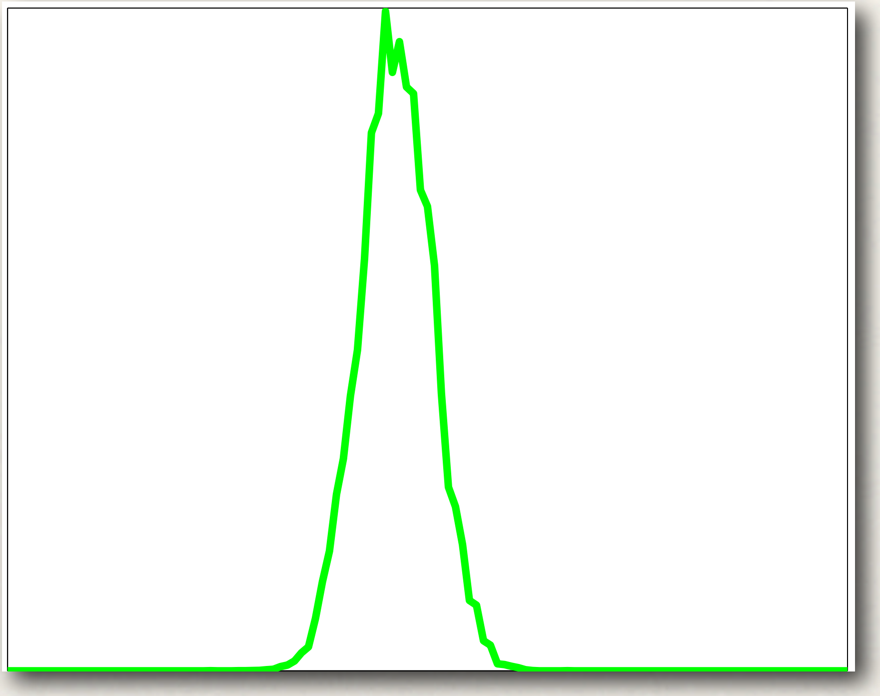
Observation *Prior*



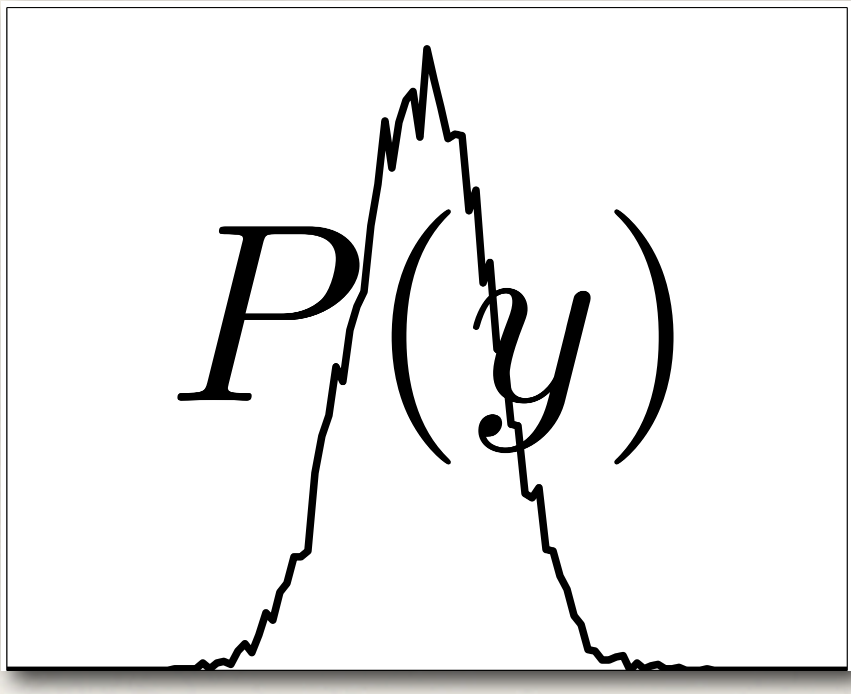
Model *Prior*



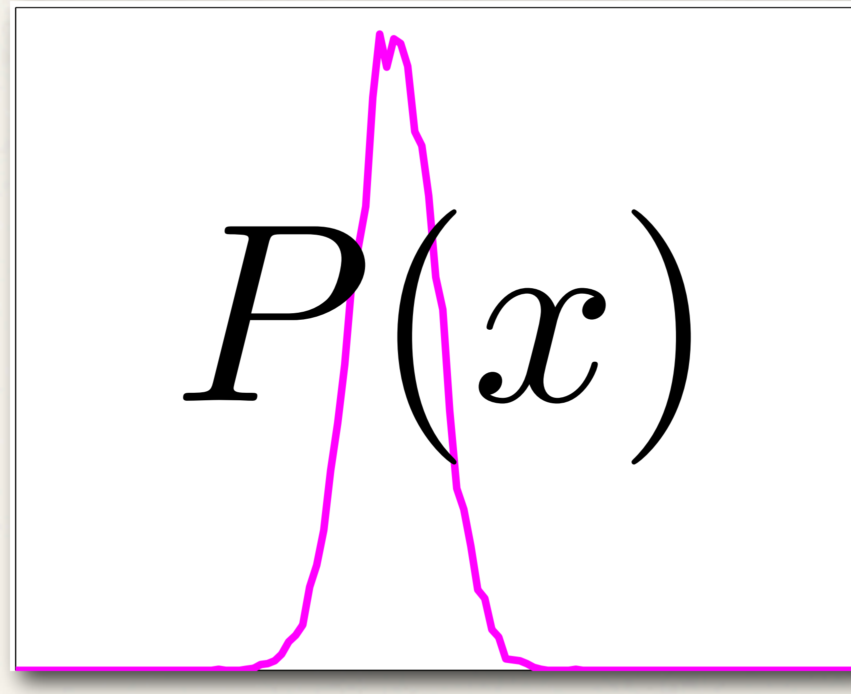
Posterior



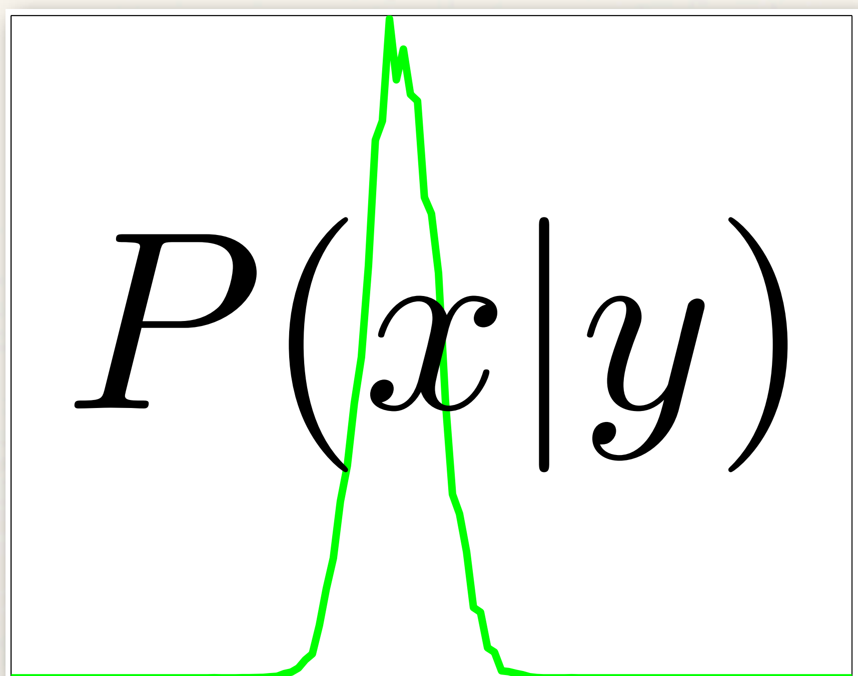
Observation *Prior*



Model *Prior*



Posterior



Rev. Thomas Bayes, 1701—1767

$$P(x|y)P(y) = P(y|x)P(x)$$

Maximum a Posterior

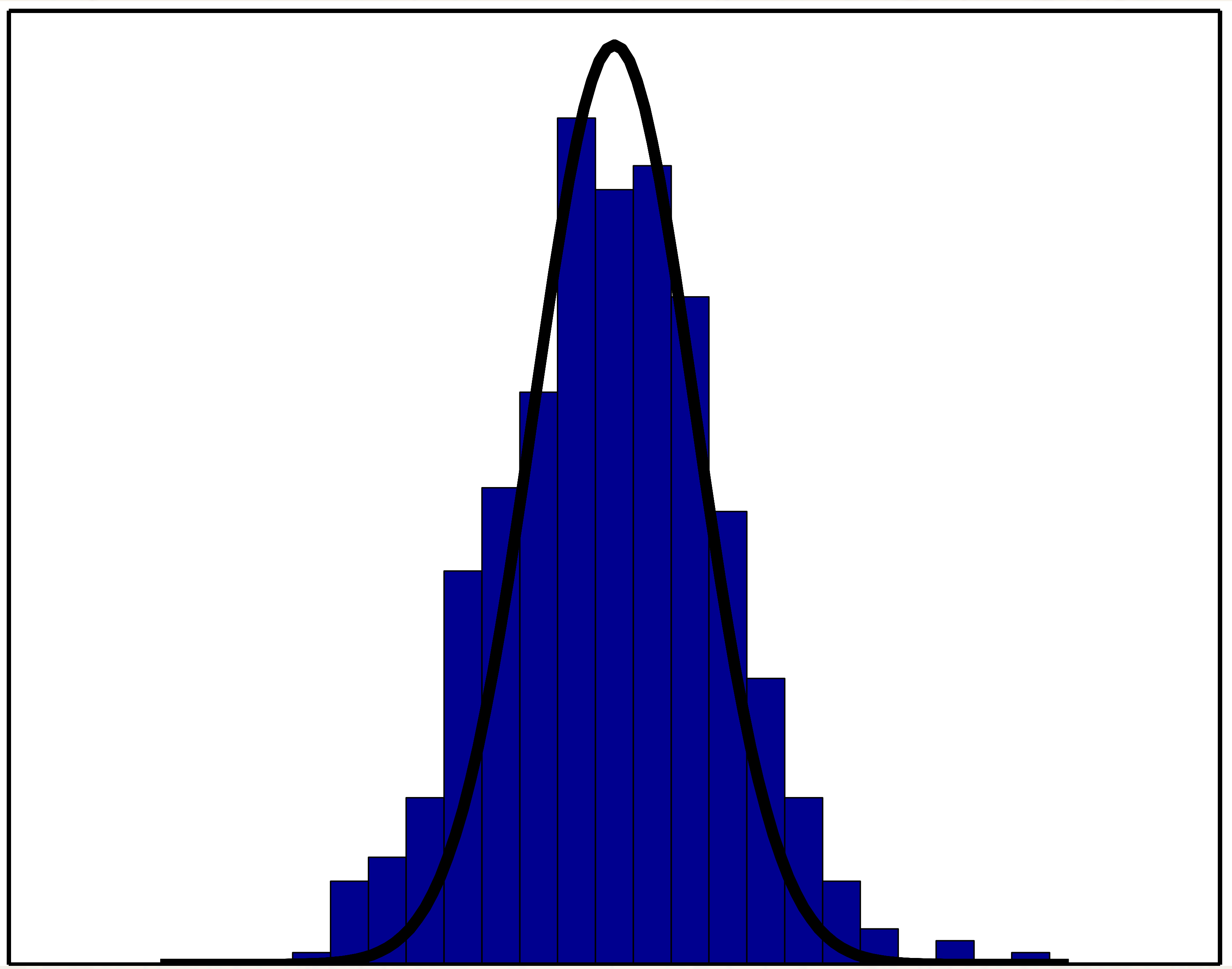
$$x_a = \max_{x \in \mathbf{X}} P(x|y)$$

$$= \max_{x \in \mathbf{X}} \frac{P(y|x)P(x)}{P(y)}$$

$$= \max_{x \in \mathbf{X}} P(y|x)P(x)$$

What PDF!?!

What PDF!?!



What PDF!?!

$$P(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-x_b)^2}{\sigma_x^2}}$$

$$P(y|x) = \frac{1}{\sigma_{yx} \sqrt{2\pi}} e^{-\frac{1}{2}\frac{(y-\mathcal{H}x)^2}{\sigma_{yx}^2}}$$

Maximum a Posterior (Max Likelihood)

$$\phi(x, y) = \ln P(x|y)$$

$$= \max_{x \in \mathbf{X}} \left(-\frac{1}{2} \frac{(x - x_b)^2}{\sigma_x^2} - \frac{1}{2} \frac{(y - x)^2}{\sigma_{yx}^2} \right)$$

$$= \min_{x \in \mathbf{X}} \left(\frac{1}{2} \frac{(x - x_b)^2}{\sigma_x^2} + \frac{1}{2} \frac{(y - x)^2}{\sigma_{yx}^2} \right)$$

Many Observations?

Vectors (many x and y)

$$\mathbf{d} = \mathbf{y} - \mathbf{H}\mathbf{x}_b$$

$$\hat{\mathbf{x}} = \mathbf{x} - \mathbf{x}_b$$

$$\mathcal{J} = \frac{1}{2}\left(\mathbf{d} - \mathbf{H}\hat{\mathbf{x}}\right)^T \mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\hat{\mathbf{x}}\right) + \frac{1}{2}\hat{\mathbf{x}}^T \mathbf{B}^{-1}\hat{\mathbf{x}}$$

Prior Errors

- $\mathbf{B}$ — Covariance of the model ocean

$$[M \times M]$$

- $\mathbf{R}$ — Covariance of the model represented observations

$$[N \times N]$$

Minimum

$$\frac{\partial \mathcal{J}}{\partial \hat{\mathbf{x}}} = 0$$

$$\therefore \quad \hat{\mathbf{x}} = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R} \right)^{-1} \mathbf{d}$$

Recall:

$$x_a = x + \alpha \left(y - \mathcal{H}x \right)$$

Solution

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}\mathbf{d}$$

where,

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1}$$

Solution

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}\mathbf{d}$$

where,

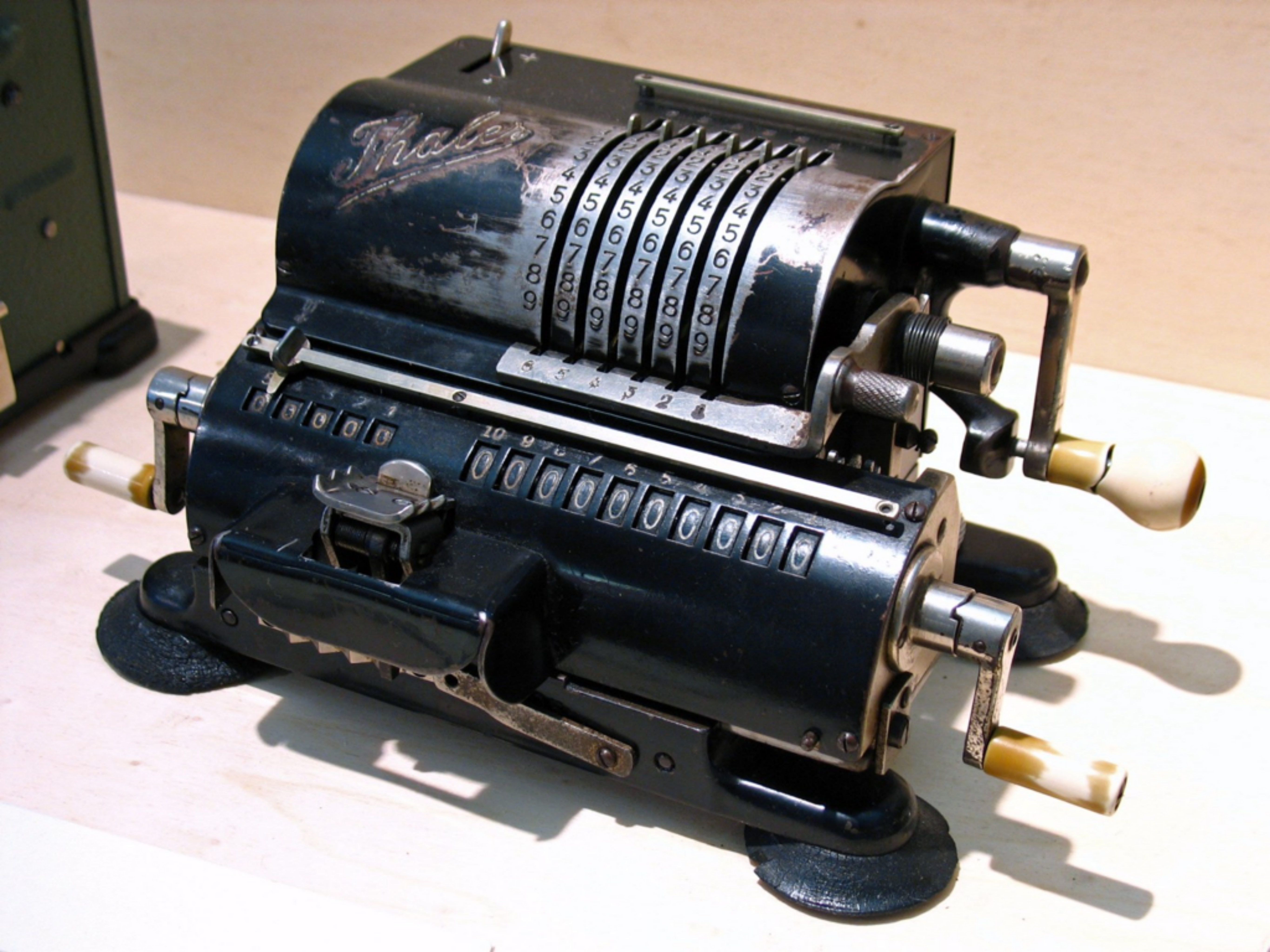
$$\mathbf{K} = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1}$$


FINISH
START

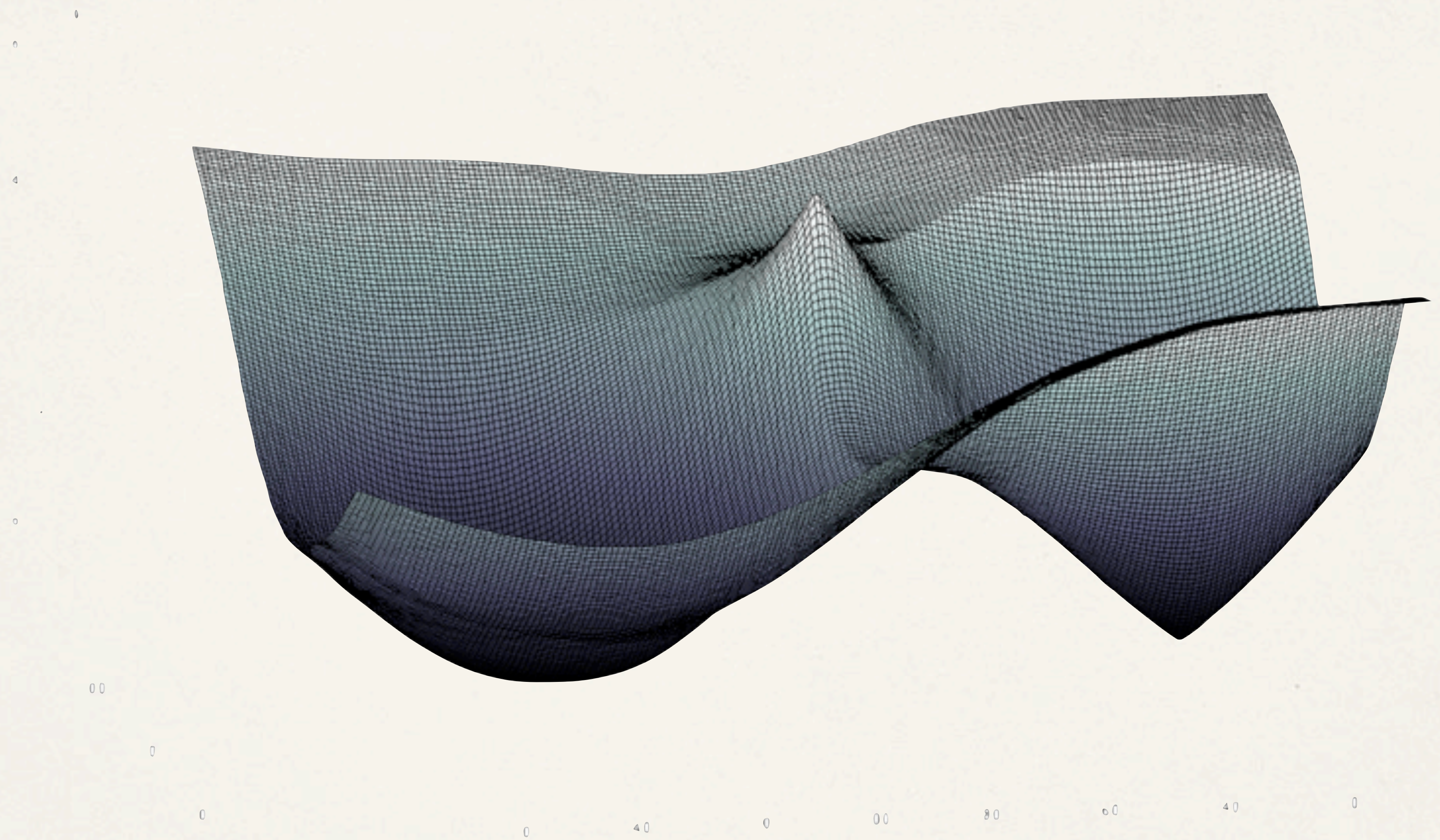

START
FINISH

$$\mathbf{K}$$

$$[M \times N]$$


Thales

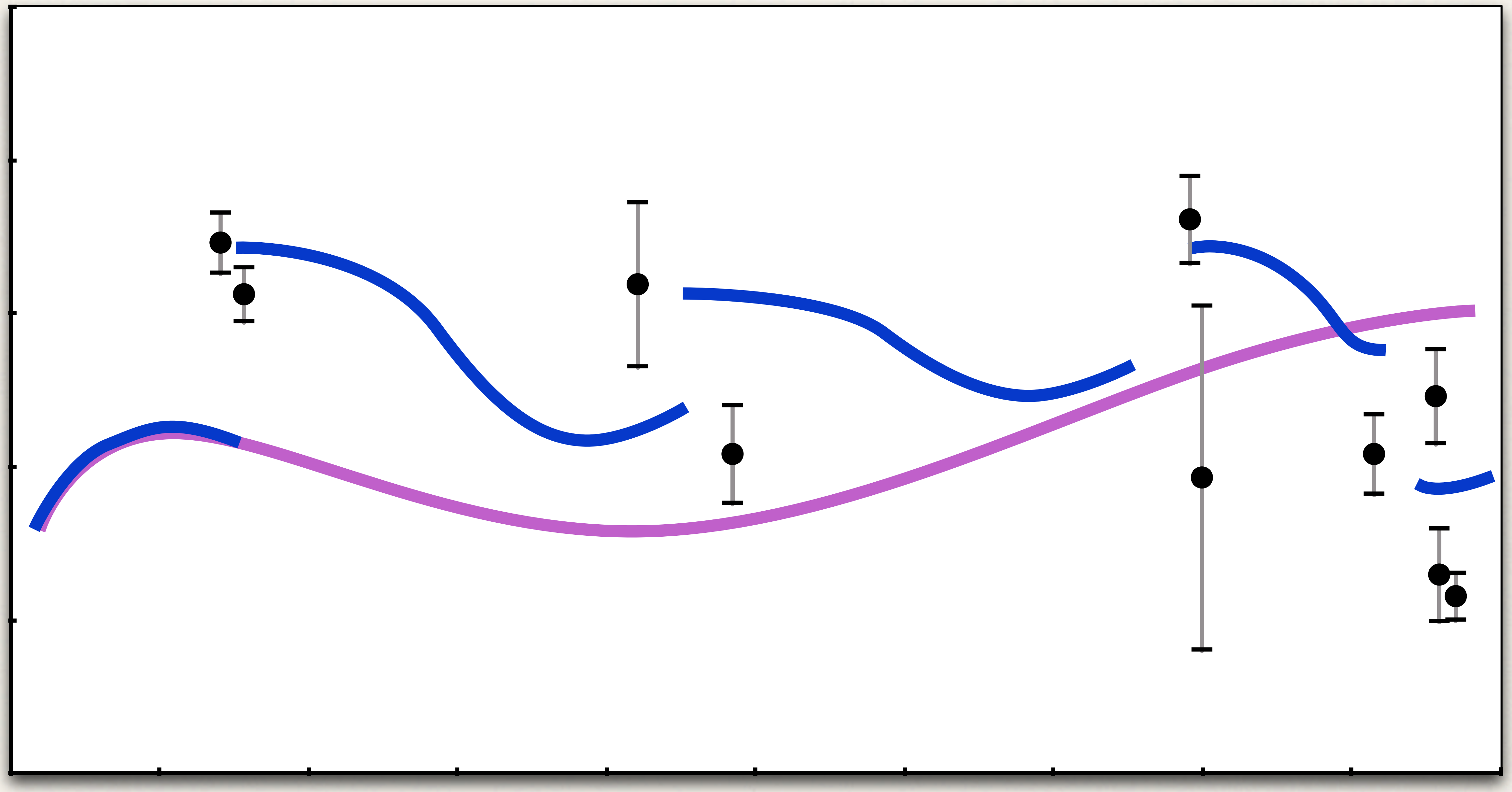
Estimate **K** via minimization



Optimal Interpolation, Kriging, 3D-Var

$$\mathbf{x}_a - \mathbf{x}_b = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1} \mathbf{d}$$

- Assume $\mathbf{B}$ is non-varying
- Observations are synoptic
- Can usually solve $\mathbf{K}$ directly



Monte-Carlo: Ensemble Kalman Filter

$$\mathbf{x}_a - \mathbf{x}_b = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1} \mathbf{d}$$

- No longer assume synoptic, non-varying
- Propagate increments with NLM

$$\mathcal{N}(\hat{\mathbf{x}} + \mathbf{x}_b)$$

- Update prior error with statistics

$$\mathbf{B}_t = \left\langle (\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T \right\rangle$$

- "Frequentist" Approach: did you sample enough?
- Minimization still in linear space

4D-Var

$$\mathbf{x}_a - \mathbf{x}_b = \mathbf{B}\mathbf{G}^T \left(\mathbf{G}\mathbf{B}\mathbf{G}^T + \mathbf{R}\right)^{-1} \mathbf{d}$$

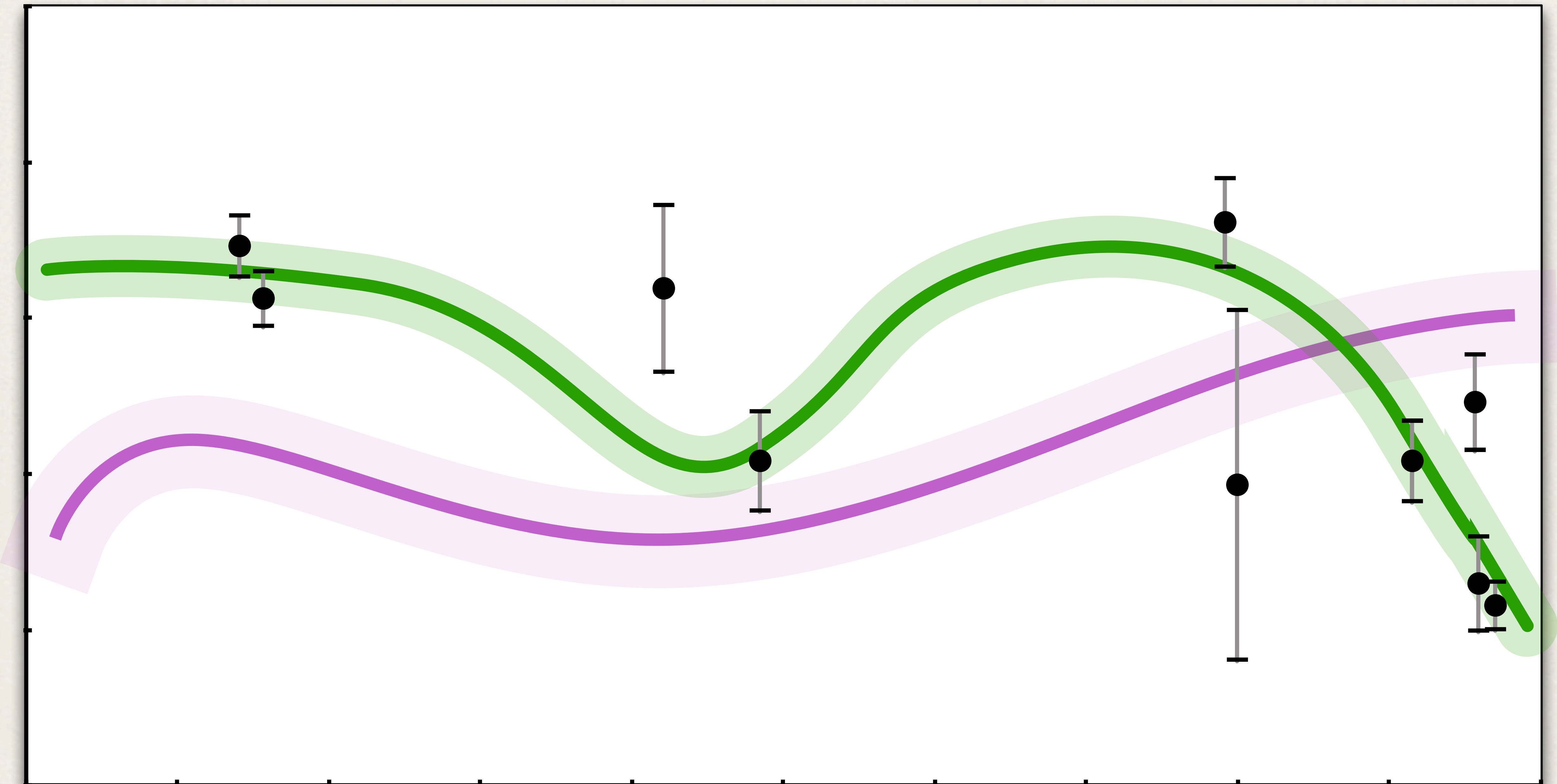
- Propagate the increments with linearized model

$$\mathbf{G}\hat{\mathbf{x}} \text{ , where } \quad \mathbf{G} = \mathbf{H}\mathbf{M}$$

- Propagate prior error with dynamics

$$\mathbf{B}_t = \mathbf{G}^{\mathbf{T}}\mathbf{B}\mathbf{G}$$

- "Solve" the problem, but in linear-space



ROMS: 4D-Var

- #define IS4DVAR
- #define W4DVAR
- The choice is to minimize in data- or model-space.
- Functionally equivalent; however, post-processing tools are plentiful in W4DVAR

ROMS 4D-Var

- To run:
  - Establish $\mathbf{B}$
  - Setup Observations, $\mathbf{y}$ and $\mathbf{R}$ [temperature, salinity, velocity, zeta, pressure, velocity radials, acoustic travel-time]
  - Configure minimization parameters
  - Experiment

ROMS: 4D-Var

- Gulf of Maine: R. He, K. Chen, B. Powell, A. Moore
- NJ/NY Coast: J. Wilkin, J. Zavala-Garay, *et al.*
- N. Atlantic: B. Powell, A. Moore
- Intra-Americas Sea: B. Powell, *et al.*
- S. California Bight: E. Di Lorenzo, A. Miller, *et al.*
- California Coastal System: A. Moore, C. Edwards, *et al.*
- Hawaii: B. Powell, *et al.*
- Philippine Sea: B. Powell, *et al.*
- Philippine Archipelago: J. Wilkin, J. Levin, *et al.*
- East Australian Current: J. Wilkin, J. Zavala-Garay