Bathymetry smoothing in ROMS: A new approach

Mathieu Dutour Sikirić
Ivica Janeković
Milivoj Kuzmić
Institute Rudjer Bosković, Zagreb

PLAN

I Problem set up

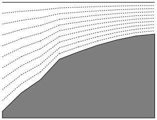
II Solution approaches

III Comparison of selected methods

I. Problem set up

Sigma coordinate systems

- One way to deal with varying bathymetry: use σ-coordinates (Phillips 1957)



- On every cell e of bathymetry $h(e)$, choose a number N of vertical levels $h(e,k)$ for $1 \leq k \leq N$ with $h(e,0) = h(e)$ and $h(e,N) = 0$.
- The differentiation rule of functions in σ-coordinate is

$$\left.\frac{\partial f}{\partial x}\right|_z = \left.\frac{\partial f}{\partial x}\right|_\sigma + \frac{\partial h}{\partial x}\frac{\partial f}{\partial \sigma}$$

- This creates a problem for horizontal derivatives, which become a difference of two terms. The wrong computation of the horizontal pressure gradient creates artificial currents.
- Smagorinsky 1967, Janjić 1977, Mesinger 1982, Haney 1991

The roughness factors

- If e and e' are two adjacent wet cells, then

$$rx_0(h, e, e') = \frac{|h(e) - h(e')|}{h(e) + h(e')}$$

The maximum over all such pairs is $rx_0(h)$, i.e. the Beckman & Haidvogel number.

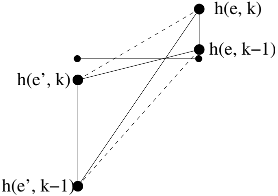
- If the vertical levels of the bathymetries are $h(e, k)$ for $1 \leq k \leq N$ then

$$rx_1(h, e, e', k) = \frac{|h(e,k) - h(e',k) + h(e,k-1) - h(e',k-1)|}{h(e,k) + h(e',k) - h(e,k-1) - h(e',k-1)}.$$

The maximum over k and pairs e, e' of adjacent wet cells is $rx_1(h)$.
This number is named hydrostatic instability number or Haney number.

Hydrostatic stability

- Denote by $C_k(e)$ the parallelepiped of water between depth $h(e, k-1)$ and depth $h(e, k)$.
- Hydrostatic stability means that if e and e' are any two adjacent cells, then $C_k(e)$ and $C_k(e')$ share a level.



- To impose that $C_k(e)$ and $C_k(e')$ share a level is equivalent to $rx_1(h, e, e', k) \leq 1$ (Rousseau and Pham 1971, Mesinger 1982, Haney 1991).
- This requirement is very strong and almost impossible to fulfill.

Vertical parametrization in ROMS

- The ROMS vertical parametrization depends on three parameters hc, θ_s, θ_b

$$h(e,k) = s_w(k)hc + (h(e) - hc)C_w(k).$$

hc is the thermocline parameter and it is lower than the minimal depth of the model.

- The vertical parametrization function depends on θ_s and θ_b and is $s_w(k) = \frac{N-k}{N}$.

$$C_w(k) = (1-\theta_b)\frac{\sinh \theta_s s_w(k)}{\sinh \theta_s} + \theta_b \left\{ \frac{1}{2} + \frac{\tanh \theta_s (s_w(k) - \frac{1}{2})}{2 \tanh \frac{\theta_s}{2}} \right\}$$

This formula is relatively arbitrary (Song 1994) and another one may work just as well.

- If $hc = 0$ then we have $h(e,k) = h(e)C_w(k)$ and we get

$$rx_1(h) = \max_{1 \le k \le N} \frac{C_w(k) + C_w(k-1)}{C_w(k) - C_w(k-1)} rx_0(h)$$

What are the right parameters?

There is no general agreement on this question

- The parameters θ_s, θ_b have to be chosen to represent correctly the vertical structure.
- The factor which matters for the horizontal pressure gradient is the Haney number $rx_1(h)$.
- It is extremely difficult to achieve $rx_1(h) \leq 1$.
- In Mellor-Ezer-Oey 1994 it is argued that the HPG error is not very important and disappears after running the model for some time.
- Kliem-Pietrzak, 1999 contests this for the Skagerrak region.
- Sasha Shchepetkin, 2008 says that $rx_1(h) \leq 3$ is "safe", $rx_1(h) \simeq 5$ is "common" and $rx_1(h) \geq 8$ is "insane".
- Kate Hedstrom, 2008 reported no problem with $rx_1(h) \simeq 16$.
- We experienced blow ups with grids with $rx_1(h) \geq 10$.

Possible ways to deal with the problem

If the bathymetry is too steep then this causes instabilities and inaccuracies. Some possible ways to deal with it:

- Use a high order pressure gradient scheme (Chu & Fan, 1997, 1998, 2003) (computational price)
- Adjust the vertical stratification, i.e. s_w, C_w and in case of ROMS θ_s, θ_b.
- Decrease the number N of vertical levels (less realistic)
- Make the horizontal grid finer (computational price).
- Smooth the bathymetry (less realistic).
- Use a z- or generalized coordinate system (change of model).

We consider the smoothing methods to reduce the magnitude of the problem.

II. Solution approaches

The goal

- The grid is build in the following way:
  - Build an initial grid using coastline informations.
  - Choose the parameters θ_s, θ_b and hc.
  - Interpolate the initial bathymetry h^{obs} from existing data set (NOAA, Gshhs, Gebco, etc.)
  - Determine the smoothed bathymetry h.
- Requirements:
  - $rx_0(h)$ and $rx_1(h)$ low.
  - The "distance" between h and h^{obs} small.
  - h should have the same physical characteristics as h^{obs}.
- For a given r and h^{obs}, we will present methods to get h with $rx_0(h) \leq r$.
- The analysis for rx_1 works similarly.

The Shapiro filter

- It is a filter designed to smooth out fast waves in finite difference models (Shapiro 1975).
- It is applied to the bathymetry in the following way:

$h \leftarrow h^{obs}$
while $rx_0(h) > r$ **do**
  $h' \leftarrow$ Shapiro filtering of h on x direction.
  for e in wet cells **do**
    if $rx_o(h, e) > r$ **then**
      $h(e) \leftarrow h'(e)$
    end if
  end for
  Do the same in y direction
end do

- For some bathymetries the Shapiro filter converges to h with $rx_0(h) > r$ and thus the program never ends.

Laplacian filter

- It works in the following way:
  - start with $h = h^{obs}$.
  - If $rx_0(h, e) > r$ we do:

$$h(e) \leftarrow h(e) + \frac{1}{2N(e)} \sum_{e' \in N(e)} \{h(e') - h(e)\}$$

    with $N(e)$ the set of wet cells adjacent to the wet cell e.
  - Iterate until $rx_0(h) \leq r$.
- This filter is more stable than Shapiro filter, but there is a still a problem of having the program end.
- Shapiro filter and Laplacian filter are very frequently used but they are not very good methods.

The Martinho & Batteen scheme

- Whenever the the roughness is not correct the chose solution (Martinho & Batteen 2006) is to increase the bathymetry.
  - Start with $h = h^{obs}$
  - If
  $$\frac{h(e) - h(e')}{h(e) + h(e')} > r \quad \text{then} \quad h(e') \leftarrow \frac{1-r}{1+r} h(e)$$
  - All pairs (e, e') are considered iteratively until the slope factor is correct. The result is independent of the order of operations.
- They also proposed to preserve the volume by replacing the bathymetry h obtained by their method by
$$h \leftarrow h \frac{\text{vol } h^{obs}}{\text{vol } h}$$
- Note: the strategy of reducing the bathymetry does not work efficiently.

The Mellor-Ezer-Oey scheme

- (Mellor 1994) If we want to preserve volume, then another scheme is possible.
  - If we have
    $$\frac{h(e) - h(e')}{h(e) + h(e')} > r$$
    then we write
    $$h(e) \leftarrow h(e) - \delta \quad \text{and} \quad h(e') \leftarrow h(e') + \delta$$
    with δ adjusted so that $\frac{h(e)-h(e')}{h(e)+h(e')} = r$.
  - All pairs (e, e') of adjacent wet cells are considered iteratively until the bathymetry is correct.
- A priori, the final bathymetry depends from the order of the operations.

Linear programming methods

- The inequality $rx_0(h, e, e') \leq r$ corresponds to two linear inequalities:

$$-r(h(e) + h(e')) \leq h(e) - h(e') \leq r(h(e) + h(e'))$$

- We introduce some auxiliary variable $\delta(e)$ with

$$|h(e) - h^{obs}(e)| \leq \delta(e) \quad \text{i.e.} \quad \pm (h(e) - h^{obs}(e)) \leq \delta(e)$$

- And we minimize

$$\sum_e \delta(e) \quad \text{that is} \quad \sum_e |h(e) - h^{obs}(e)|.$$

- There are many possible variants, which are still in the linear programming paradigm:
  - Preserve the total volume of the basin.
  - Have a different objective function.
  - Impose only positive/negative corrections at some points.
  - Impose maximum amplitude condition.

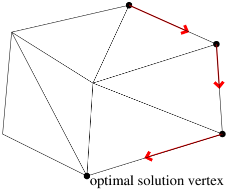
Definition

- A linear program is the problem of maximizing a linear function $f(x)$ over a set $\mathcal{P}$ defined by linear inequalities (polyhedral).

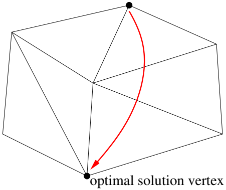
$$\mathcal{P} = \{x \in \mathbb{R}^d \quad \text{such that} \quad f_i(x) \geq b_i\}$$

with f_i linear and $b_i \in \mathbb{R}$.

- The solution of linear programs is attained at vertices of $\mathcal{P}$.
- There are two classes of solution methods:



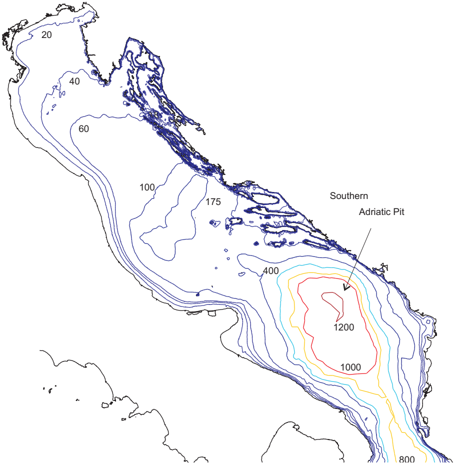
Simplex method



Interior point method

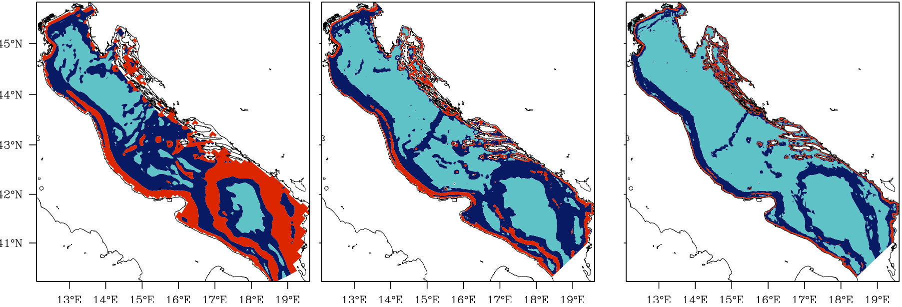
III. Comparison of selected methods

The Adriatic Sea



- The bathymetry is highly varying and the coastline is diverse.
- We chose three grids 160×60, 127×368, 271×751

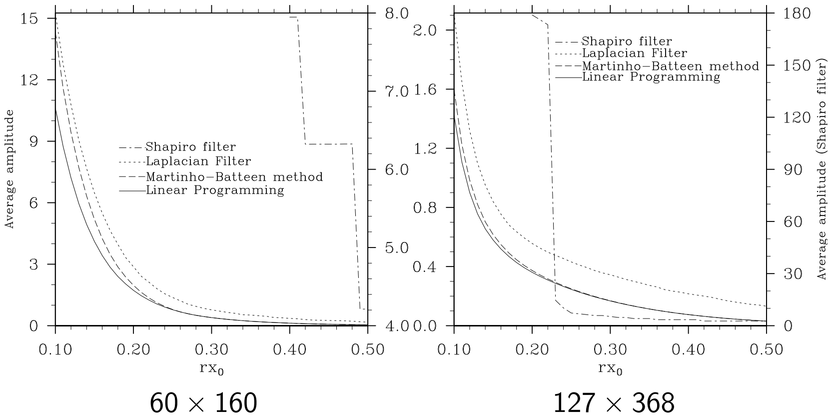
Regions of hydrostatic stability



60×160 127×368 271×751

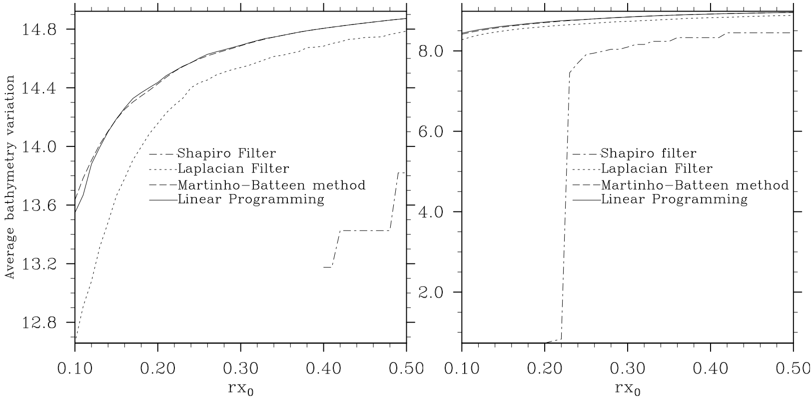
The regions of hydrostatic consistency ($rx_1(h, e) \leq 1$ in light blue), hydrostatic inconsistency ($1 \leq rx_1(h, e) \leq 5$ in dark blue) and hydrostatic instability ($rx_1(h, e) \geq 5$ in red)

Average amplitude of bathymetry modification



The average amplitude of bathymetry modification (m) in terms for bathymetry smoothing methods

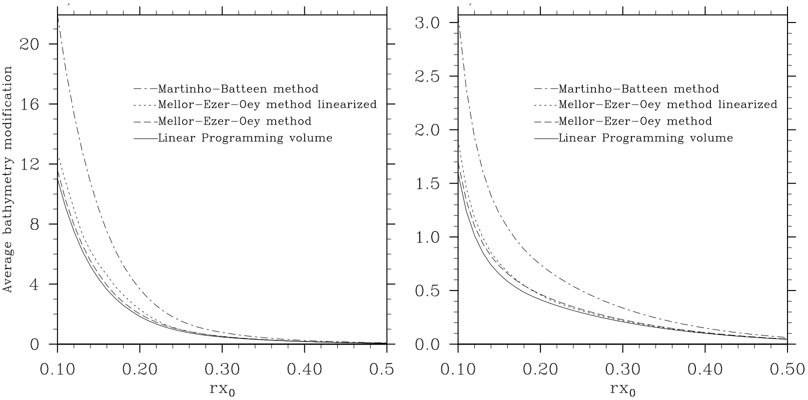
Average variation of bathymetry



60×160 127×368

The average variation of the bathymetry (m) from wet cell to wet cell for bathymetry smoothing methods in terms of $rx_0(h)$

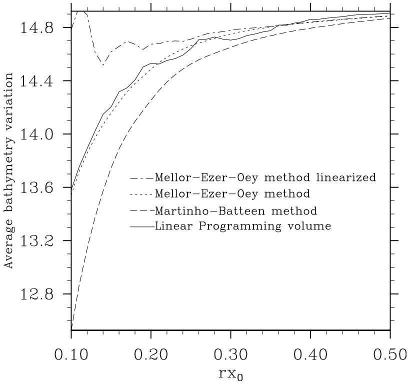
Average amplitude of bathymetry modification



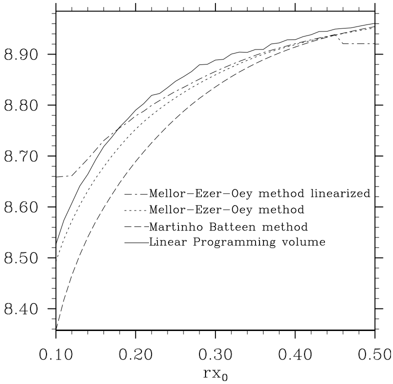
60×160 127×368

The average amplitude of bathymetry modification (m) in term of rx_0 for volume preserving smoothing methods

Average variation of the bathymetry



60×160



127×368

The average variation of the bathymetry (m) from wet cell to wet cell for bathymetry smoothing methods preserving volume in terms of rx_0

Stability of solutions

What happens if one perturb by an infinitesimal quantity the observed bathymetry and/or the roughness factor?

- Heuristic methods are continuous.
- Shapiro filter and Laplacian filter methods are not continuous.
- Linear programming methods are not continuous since there are possible hoppings from one vertex to an adjacent one.

In practice during 0.01 increments to rx_0,

- Shapiro/Laplacian filter are 10 times more discontinuous than heuristic method.
- polyhedral method are 2 times more discontinuous.

Conclusions

- Shapiro and Laplacian filter should be avoided since they create large perturbation of the bathymetry.
- Heuristic methods like Martinho-Batteen, Mellor-Ezer-Oey work very well.
- If $rx_0(h) \leq 0.2$ is needed, then linear programming might be what you need.
- All programs for optimizing over rx_0 or rx_1 are available from http://drobilica.irb.hr/~mathieu/Bathymetry/index.html

THANK

YOU