Ocean-Atmosphere coupling

towards a mathematically and physically consistent algorithm

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Context

- **Two-way coupling** between realistic regional or global numerical models simulating oceanic and atmospheric circulations.

- What is the **legitimacy** of the coupling methods used presently from the **mathematical** and **physical** point of view ?

  This question has been very few studied in the literature.

  Which **future improvements** can we expect ?

- **Seen in a few papers :**

  ~~The Coupling Scheme: OASIS~~

  OASIS (and MCT) is **NOT** a coupling scheme, this is a software to facilitate the implementation of a given coupling scheme !

Context

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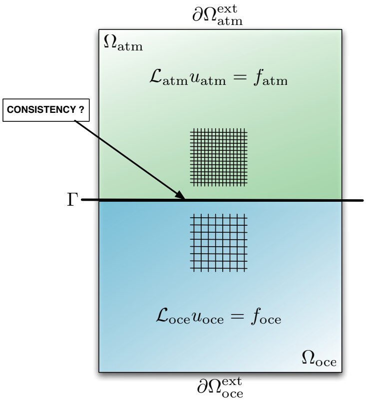
  This question has been very few studied in the literature.

  Which **future improvements** can we expect ?

- useful for several reasons
  - effect of orography in coastal regions
  - better resolution of the diurnal cycle
  - better representation of the interactions between OBL and ABL
  - etc ...

Formalism of the problem



- *oceanic model:* $\mathcal{L}_{oce}$
- *atmospheric model:* $\mathcal{L}_{atm}$
- *state variables:* u_{atm} and u_{oce}

The coupling problem under interest reads :

Find u_{oce} and u_{atm} that satisfies

$$\begin{array}{lcll} \mathcal{L}_{oce}u_{oce} & = & f_{oce} & \text{in } \Omega_{oce} \times [0,T] \\ \mathcal{B}_{oce}u_{oce} & = & g_{oce} & \text{in } \partial\Omega_{oce}^{ext} \times [0,T] \\ & & & \\ \mathcal{L}_{atm}u_{atm} & = & f_{atm} & \text{in } \Omega_{atm} \times [0,T] \\ \mathcal{B}_{atm}u_{atm} & = & g_{atm} & \text{in } \partial\Omega_{atm}^{ext} \times [0,T] \end{array}$$

+ a given consistency on $\Gamma \times [0,T]$

The problem must be completed by appropriate boundary conditions on $\Gamma \times [0,T]$

Simplified problem

For our theoretical study we propose to work on a simplified problem :

$$\mathcal{L}_{\text{atm}} = \frac{\partial}{\partial t} - \frac{\partial}{\partial z}\left(K_z^a(z)\frac{\partial}{\partial z}\right) \qquad \mathcal{L}_{\text{oce}} = \frac{\partial}{\partial t} - \frac{\partial}{\partial z}\left(K_z^o(z)\frac{\partial}{\partial z}\right)$$

(equations for vertical diapycnal transport of a tracer or velocity component)

- diffusion equations (with spatially variable coefficients) are assumed to satisfactorily describe the effect of turbulent mixing in the boundary layer
- the diffusion profiles $K_z^a(z)$ and $K_z^o(z)$ are determined by a *K-profile* closure model (like *Troen and Mahrt* scheme or *KPP* scheme).
- Those equations are directly forced by air-sea fluxes.

This simplified problem is representative of the "true" coupling problem and contains a large part of the difficulties.

Natural boundary conditions

More generally we look for the solution which satisfies the physical transmission conditions (continuity of the solution and of its normal derivative on the interface)

- For the first part of this talk we make the assumption that the boundary layers are explicitely resolved

Ideally, the solution u of the coupled problem should satisfy

$$u_{\mathrm{atm}}(0,t) = u_{\mathrm{oce}}(0,t) \qquad K_z^a \frac{\partial u_{\mathrm{atm}}}{\partial z}(0,t) = K_z^o \frac{\partial u_{\mathrm{oce}}}{\partial z}(0,t)$$

Required consistency = continuity of the solution and the normal component of its flux across the internal interface !

How to impose this consistency ?

We are looking for a new algorithmic approach... but under some constraints

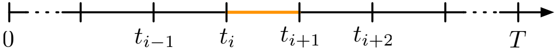
- Affordable from the computational cost point of view.
- Not only a mathematical problem, physical parameterizations must be rigorously taken into account $\Rightarrow$ **this is a crucial point.**
- Easy management of non-conformity in space and time.
- Minimal modifications of existing numerical codes.

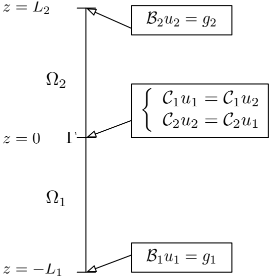
Optimized Global-in-Time Schwarz Methods appear to be a good compromise + a rich and rigourous mathematical framework

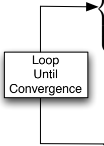
Global-in-time Schwarz method

We temporarily remove the subscript *atm* and *oce*

Iterative process : first guess $u_2^0(0,t)$







$$
\begin{cases}
\mathcal{L}_1 u_1^k = f, & \text{in } \Omega_1 \times [t_i, t_{i+1}], \\
u_1^k(z, t = t_i) = u_0(z), & z \in \Omega_1, \\
\mathcal{B}_1 u_1^k(-L_1, t) = g_1, & t \in [t_i, t_{i+1}], \\
\mathcal{C}_1 u_1^k(0,t) = \mathcal{C}_1 u_2^{k-1}(0,t), & \text{in } \Gamma \times [t_i, t_{i+1}],
\end{cases}
$$

$$
\begin{cases}
\mathcal{L}_2 u_2^k = f, & \text{in } \Omega_2 \times [t_i, t_{i+1}], \\
u_2^k(z, t = t_i) = u_0(z), & z \in \Omega_2, \\
\mathcal{B}_2 u_2^k(L_2, t) = g_2, & t \in [t_i, t_{i+1}], \\
\mathcal{C}_2 u_2^k(0,t) = \mathcal{C}_2 u_1^k(0,t), & \text{in } \Gamma \times [t_i, t_{i+1}],
\end{cases}
$$

We must choose the **transmission conditions** $\mathcal{C}_1$ and $\mathcal{C}_2$ to match the required consistency and to **optimize the convergence speed**.

Optimized Global-in-time Schwarz method

First, let's consider the coupling of two diffusion equations with **constant and discontinuous coefficients** (e.g. molecular viscosities)

$$\mathcal{L}_j = \frac{\partial}{\partial t} - \nu_j \frac{\partial^2}{\partial z^2}, \qquad (j = 1, 2)$$

we intend to impose the **continuity of the solution** and of the **normal components of the fluxes** across Γ

$$u_1(0,t) = u_2(0,t), \qquad \nu_1 \frac{\partial u_1}{\partial z}(0,t) = \nu_2 \frac{\partial u_2}{\partial z}(0,t)$$

easiest way to impose it : **Dirichlet-Neumann** type algorithm

$$\begin{cases} \mathcal{L}_1 u_1^k = f & \text{in } \Omega_1 \times [t_i, t_{i+1}] \\ u_1^k(0,t) = u_2^{k-1}(0,t) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

$$\begin{cases} \mathcal{L}_2 u_2^k = f & \text{in } \Omega_2 \times [t_i, t_{i+1}] \\ \nu_2 \dfrac{\partial u_2^k}{\partial z}(0,t) = \nu_1 \dfrac{\partial u_1^k}{\partial z}(0,t) & \text{on } \Gamma \times [t_i, t_{i+1}] \end{cases}$$

Optimized Global-in-time Schwarz method

Convergence factor for a **Dirichlet-Neumann** algorithm

$$\rho_{DN} = \left| \frac{e^{k+1}}{e^k} \right| = \sqrt{\frac{\nu_2}{\nu_1}}$$

1. ρ_{DN} is not always smaller than one i.e. for certain values of the **diffusion coefficients** the algorithm is **divergent**
2. **slow convergence speed** if the discontinuity between ν_1 and ν_2 is small.

Now let's suppose a **new kind of transmission conditions** of **Robin (Fourier)** type

$$\begin{cases} \nu_1 \dfrac{\partial u_1}{\partial z}(0,t) + \Lambda_2 u_1(0,t) & = & \nu_2 \dfrac{\partial u_2}{\partial z}(0,t) + \Lambda_2 u_2(0,t), \\[2ex] -\nu_2 \dfrac{\partial u_2}{\partial z}(0,t) + \Lambda_1 u_2(0,t) & = & -\nu_1 \dfrac{\partial u_1}{\partial z}(0,t) + \Lambda_1 u_1(0,t), \end{cases}$$

with Λ_1 and Λ_2 are two **local or nonlocal operators** (P.L. Lions, 1990).

Optimized Global-in-time Schwarz method

By introducing $\mathbf{e_j^k = u_j^k - u^\star}$ and defining the symbols $\widehat{\mathbf{\Lambda_j e_j}} = \lambda_\mathbf{j}\widehat{\mathbf{e}}_\mathbf{j}$ one gets in Fourier space

$$\rho_{RR} = \left| \frac{\widehat{e}_j^k}{\widehat{e}_j^{k-1}} \right| = \left| \frac{(\lambda_1 + \nu_2\sigma_2^-)}{(\lambda_1 + \nu_1\sigma_1^+)} \frac{(\lambda_2 - \nu_1\sigma_1^+)}{(\lambda_2 - \nu_2\sigma_2^-)} \right|, \qquad \sigma_j^\pm = \pm\sqrt{\frac{i\omega}{\nu_j}}$$

The **optimal transmission conditions** (ensuring a convergence in 2 iterations) are given by

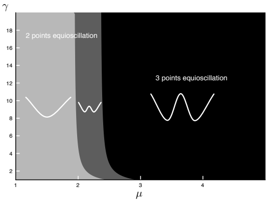
$$\lambda_1 = \sqrt{i\omega\nu_2}, \qquad \lambda_2 = \sqrt{i\omega\nu_1}, \qquad \mathcal{F}^{-1}(\lambda_j) * u_j = \int_0^{+\infty} \mathcal{F}^{-1}(\lambda_j)(\tau) u_j(x, t-\tau) d\tau.$$

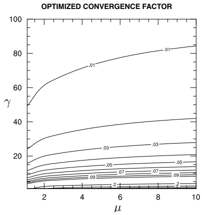
those operators are **non-local in time**, we look to approximate them by local ones (**polynomial in $i\omega$**) $\qquad \lambda_j = p_j + q_j i\omega + ...,$
the best **zeroth-order approximation** of the absorbing conditions is the solution of the minimax problem

$$\min_{p_1, p_2} \left(\max_{\omega \in [\omega_{min}, \omega_{max}]} \rho_{RR} \right)$$

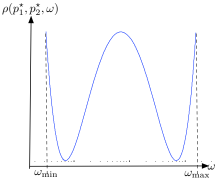
ω_{min} and ω_{max} corresponds to the minimum and maximum temporal frequencies expected during a simulation.

A few theoretical results

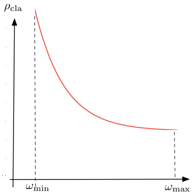




$$\gamma = \sqrt{\frac{\nu_2}{\nu_1}} \qquad \mu = \left(\frac{\omega_{\max}}{\omega_{\min}}\right)^{\frac{1}{4}}$$



Optimized Schwarz Method



Classical Schwarz Method

$\rho_{\text{cla}}(\omega_{\min}) > \rho(\omega_{\min})$

$\rho(\omega_{\min}) = \rho(\omega_{\max}) \Rightarrow$ the algorithm converges equally well for low and high frequencies

F. Lemarié, L Debreu and E. Blayo, Towards an optimized global-in-time Schwarz algorithm for diffusion equations with discontinuous and spatially variable coefficients. Part 1 : the constant coefficients case, *submitted to SIAM Journal on Scientific Computing*

Do we meet the expectations ?

- The optimization of the convergence factor makes the method robust and tractable from the computational cost point of view.

- This method requires minor modifications of existing codes: adjustments of the tridiagonal matrix for the implicit treatment of diffusion to enable the use of Robin-type conditions.

- The management of non-conformity in space and time is natural and a lot of theoretical studies have been done in the context of Schwarz-like mehods (M.J. Gander, 2003; Blayo *et al*, 2007).

Last, but not least, we said:

- "Not only a mathematical problem, physical parameterizations must be rigorously taken into account ⇒ **this is a crucial point."**

Unfortunately this is also a very complex point !

1. We extended previous results to the case with spatially variable coefficients given by **KPP-like** closure schemes.

2. We also have to be consistent with the **air-sea fluxes parameterizations.**

Reminder about the derivation of bulk formulae

The turbulent terms ($\langle u'w' \rangle_0$, $\langle \theta' w' \rangle_0$, ...) involved in the computation of air-sea fluxes are generally computed via the *bulk* formulae.

Starting from the *Reynolds averaged equations* the following assumptions (supposed valid in the atmospheric surface layer) are done

- hydrostaticity + Boussinesq assumption
- horizontal homogeneity
- quasi-stationarity
- Coriolis force and HPG are negligible

Finally we get the constant flux layer approximation :

$$\frac{\partial}{\partial z}\left[\langle u'w' \rangle + \nu \frac{\partial \langle u \rangle}{\partial z}\right] = 0 \qquad \frac{\partial}{\partial z}\left[\langle \theta' w' \rangle + \nu \frac{\partial \langle \theta \rangle}{\partial z}\right] = 0$$

by neglecting the molecular effects and by noting τ the constant flux of momentum and Q_H the constant flux of sensible heat we get

$$-\langle u'w' \rangle_0 = \frac{\tau}{\rho_a} = u_*^2 \qquad -\langle \theta' w' \rangle_0 = \frac{Q_H}{\rho_a C_{p,a}} = Q_0$$

Reminder about the derivation of bulk formulae

We look to parameterize the terms $\langle u'w' \rangle_0$ and $\langle \theta' w' \rangle_0$ in term of the mean quantity of the flow.
$\Rightarrow$ *semi empirical Monin Obukhov similarity theory*

It argues that the only important turbulence parameters in the dry surface layer are u_*, Q_0, z and $\beta = \dfrac{g}{\langle \theta \rangle}$ (the buoyancy).

Thanks to a dimensional analysis the following non dimensional π-groups are found

$$\pi_1 = \frac{\partial \langle u \rangle}{\partial z} \frac{kz}{u_*} \qquad \pi_2 = \frac{\partial \langle \theta \rangle}{\partial z} \frac{kzu_*}{Q_0} = \frac{\partial \langle \theta \rangle}{\partial z} \frac{kz}{\theta_*} \qquad \pi_3 = \frac{zk\beta Q_0}{u_*^3} = \frac{z}{L_{\mathrm{MO}}}$$

Finally we empirically find functional relations between those groups

$$\pi_1 = \phi_m(\pi_3) \qquad \pi_2 = \phi_s(\pi_3)$$

where ϕ_m and ϕ_s are universal functions (Businger *et. al.*, 1971; Hogstrom, 1988).

Reminder about the derivation of bulk formulae

Integration of the previous relations (Paulson, 1970) gives the mean quantity profiles in the surface layer, **but not below**

$$\langle u \rangle (z) = SSU + \frac{u_*}{k} \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\left(\frac{z}{L_{\mathrm{MO}}}\right)\right]$$

$$\langle \theta \rangle (z) = SST + \frac{\theta_*}{k} \left[\ln\left(\frac{z}{z_\theta}\right) - \psi_s\left(\frac{z}{L_{\mathrm{MO}}}\right)\right]$$

Thanks to those equalities we get

$$\frac{\vec{\tau}}{\rho_a} = C_D(z, z_0, L_{\mathrm{MO}})|\Delta\vec{U}|\Delta\vec{U} \qquad \frac{Q_H}{\rho_a C_{p,a}} = C_D(z, z_0, z_\theta, L_{\mathrm{MO}})|\Delta\vec{U}|\Delta\theta$$

where $\Delta\vec{U}$ and $\Delta\theta$ are the difference between the values at the first vertical level in numerical models.

First important remark :

- All this theory is valid only at a certain distance from the air-sea interface, above the direct influence of the surface roughness elements.

Inclusion of air-sea fluxes in our model problem

We suggested to impose the following conditions on Γ

$$\rho_a K_z^a \frac{\partial u_{\mathrm{atm}}}{\partial z}(0,t) = \rho_o K_z^o \frac{\partial u_{\mathrm{oce}}}{\partial z}(0,t) \qquad u_{\mathrm{atm}}(0,t) = u_{\mathrm{oce}}(0,t)$$

but the behaviour of the flow near Γ (in the surface layer) is described by a **function of parameterization** and we have a poor knowledge of what happens near the viscous sublayer.

We must formulate the problem to take this into account : in practice the **new consistency** we look to impose is thus

$$\rho_a K_z^a \frac{\partial u_{\mathrm{atm}}}{\partial z}(0,t) = \rho_o K_z^o \frac{\partial u_{\mathrm{oce}}}{\partial z}(0,t) \qquad u_{\mathrm{atm}}(0^+,t) - u_{\mathrm{oce}}(0^-,t) = \gamma_{SL}$$

where $\gamma_{SL} = F(u_*, \theta_*, z_0, z_\theta, L_{\mathrm{MO}})$.

Note that by imposing explicitely the equality of the fluxes we implicitly impose the jump conditions at the interface

The following step from a theoretical point of view : design transmission conditions of Robin type (i.e. with free parameters) consistently with the surface layer parameterization. (ongoing work)

However the **Schwarz method** "as is" (i.e. without optimization) has a lot of practical advantages

Let's suppose that we intend to impose

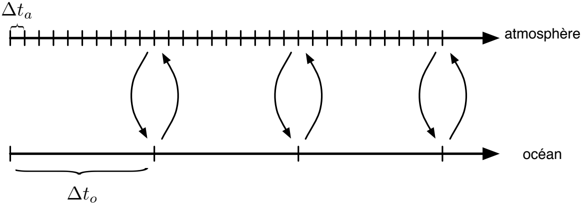
$$\left\{ \begin{array}{rcl} \rho_a K_m^a \dfrac{\partial \vec{U}_{\mathrm{atm}}}{\partial z}(0,t) & = & \rho_o K_m^o \dfrac{\partial \vec{U}_{\mathrm{oce}}}{\partial z}(0,t) = \vec{\tau} \\ \rho_a C_{p,a} K_s^a \dfrac{\partial \theta_{\mathrm{atm}}}{\partial z}(0,t) & = & \rho_o C_{p,o} K_s^o \dfrac{\partial \theta_{\mathrm{oce}}}{\partial z}(0,t) = Q_{\mathrm{net}} \end{array} \right.$$

as well as the continuity of the freshwater flux.

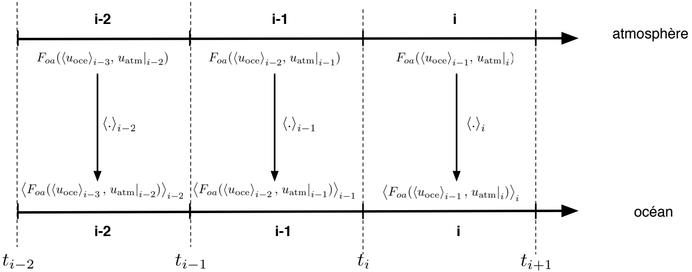
Do the current OA coupling methods address this problem satisfactorily ?

A schematic view of existing methods

- For regional applications



- For global applications : $[0, T] = \cup_{i=1}^{N} [t_i, t_{i+1}]$



Instantaneous fluxes vs averaged fluxes

B. Large, *Ocean Weather Forecasting: An Integrated View of Oceanography* :

"The sign of turbulent fluxes can be uncertain on time scales less than 10 minutes and hourly fluxes are more relevant"

"When averaging, the uncertainty should diminish as wave effects start to average out"

"The transfer coefficients C_D and C_H are determined empirically from measurements averaged over about an hour (e.g. Large and Pond 1981) and may not be applicable for instantaneous output from an atmospheric model grid point."

the choice can be to implement more detailed physical processes (generally neglected by Bulk formulations) relevant to high temporal frequency coupling : **spray contributions** to heat fluxes and the **wavy boundary layer** (Bao *et al.*, 2000).

Evaluation of existing methods

- **Coupling at the time step level :**
  - relevancy of instantaneous fluxes ?
  - the exchange of informations is done by considering an explicit treatment of surface fluxes
  - not straightforward to implement
- **Coupling by time windows :**
  - the oceanic state used to compute fluxes on a given time window is systematically delayed by one time window
  - corresponds to one (and only one) Schwarz iteration $\Rightarrow$ there is no strict equality of the fluxes (one iteration does not make the convergence)

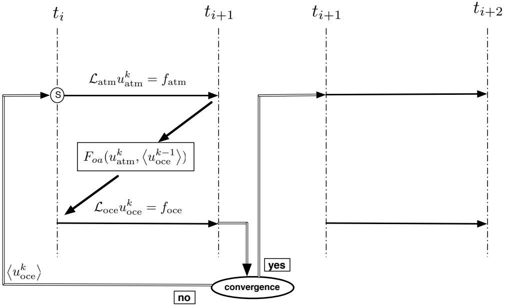
By using a **Global-in-time Schwarz algorithm** we can circumvent all these problems

A preliminary study in a real case simulation

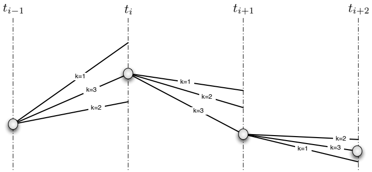
WRF / **ROMS-AGRIF** coupling : simulation of tropical cyclone Erica (2003), ANR project "Cyclones and climate" (Ch. Menkes, Nouméa)

$dx_a = 35km, dt_a = 180s, dx_o = 13km, dt_o = 1800s$

- Boundary data : ECCO, NCEP2 reanalysis
- Run coupled model for 15 days (60 x 6hours), no flux correction, no SST/SSS correction
- OA fluxes directly computed via WRF surface layer scheme (based on MOST)



Details of the implementation

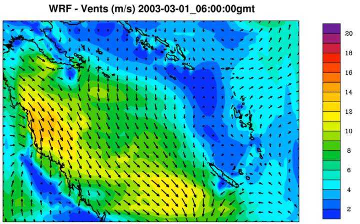


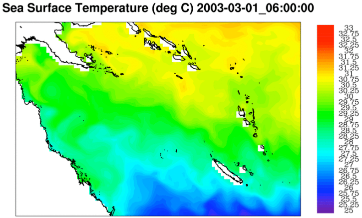
linear reconstruction of the fluxes on each time window thanks to $F(t_i)$ and $\overline{F}_{[t_i,t_{i+1}]}$

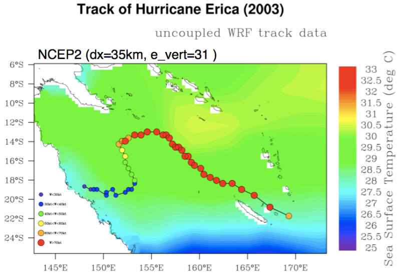
grid to grid interpolation : SCRIP package with optimizations (+ extrapolations)

For this preliminary study the exchanges between numerical models is done by files (no Message Passing).

It's important to note that's this method can be implemented by using "traditional" couplers


WRF - Vents (m/s) 2003-03-01_06:00:00gmt
20
18
16
14
12
10
8
6
4
2


Sea Surface Temperature (deg C) 2003-03-01_06:00:00


Track of Hurricane Erica (2003)
uncoupled WRF track data
NCEP2 (dx=35km, e_vert=31)
6°S
8°S
10°S
12°S
14°S
16°S
18°S
20°S
22°S
24°S
145°E
150°E
155°E
160°E
165°E
170°E
33
32.5
32
31.5
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27
26.5
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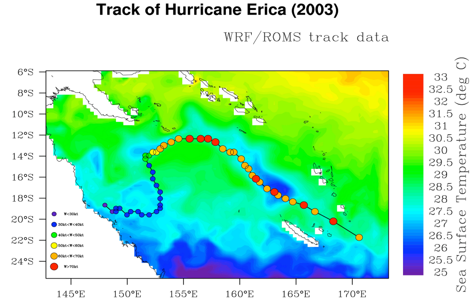
25.5
25
Sea Surface Temperature (deg C)


Track of Hurricane Erica (2003)
WRF/ROMS track data
6°S
8°S
10°S
12°S
14°S
16°S
18°S
20°S
22°S
24°S
145°E
150°E
155°E
160°E
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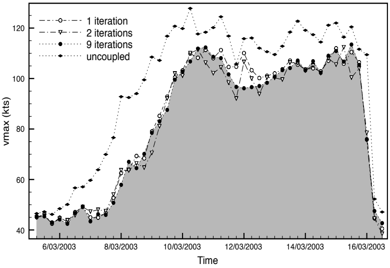
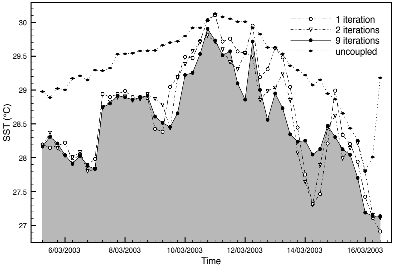
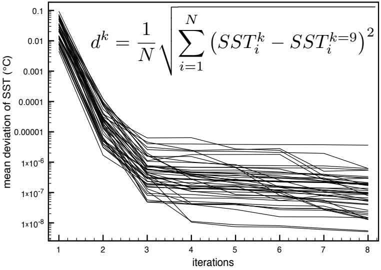
26
25.5
25
Sea Surface Temperature (deg C)

Alongtrack Vmax variations (kts)



Alongtrack SST variations (°C)





Summary and expectations

We suggest a new algorithmic framework for OA coupling

- **an iterative process** which improves **the consistency** of the coupled solution and which can be theoretically accelerated via the resolution of an **optimization problem**
- a quite **easy to implement** method :
  - minor modifications of existing codes.
  - on parallel architectures communications between both models are required only one time per time windows.

Strong qualities :

- Optimized Schwarz Methods use transmission conditions adapted to the underlying PDE, which greatly improve their convergence rate.
- A theoretical framework which could assess the compatibility between physical parameterizations of OBL and ABL.

Strong drawback : the use of an iterative process

Summary and expectations

- This is a first step towards efficient transmission conditions but we need to enforce the link with the surface layer parameterization.

  Objective : be able to obtain the best possible approximation of the converged solution with 2 iterations (optimal control theory)

- We must assess the improvements brought to the physical results.

  Objective : implementation of a simplified test-case to facilitate the intercomparison with other methods.

There is a **strong need** of this kind of testcase !

Open problems :

- initialization and data assimilation in coupled model
- coupling and nesting ?