Assimilation of images in numerical models

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Team - project INRIA MOISE

French Research Agency project ADDISA

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http://addisa.gforge.inria.fr

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Data Assimilation

ADDISA project — Motivations

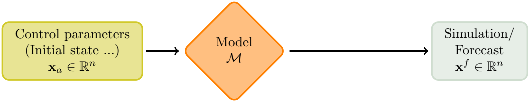
Image Assimilation : pseudo observations techniques

Direct Assimilation of Image Sequences

Numerical Experiments

Conclusions and Future works

- Forecast is produced by integration of a model from an initial state
- Data Assimilation combines in a coherent manner all the available informations to retrieve an optimal initial state and then predict it:
  - Mathematical information : model
  - In-situ and remote measurements of the true state
  - A priori knowledges (background, errors ...)



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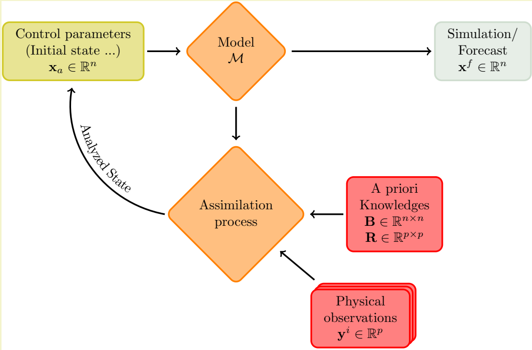

The state of the flow is governed by a partial differential system

$$\begin{cases} \dfrac{\partial \mathbf{x}}{\partial t} = \mathcal{M}(\mathbf{x}, U) \\ \mathbf{x}(0) = V \end{cases}$$

$U = U(x, y, z, t)$: unknown parameters
$V = V(x, y, z)$: initial condition

Cost function

$$J(U, V) = \frac{1}{2} \int_0^T \|\mathcal{H}[\mathbf{x}] - \mathbf{y}_{obs}\|_{\mathcal{O}}^2 + \frac{1}{2}\|U - U_0\|_{\mathcal{U}}^2 + \frac{1}{2}\|V - V_0\|_{\mathcal{V}}^2$$

Adjoint (backward) model

$$\begin{cases} \dfrac{\partial \mathbf{p}}{\partial t} + \left[\dfrac{\partial \mathcal{M}}{\partial \mathbf{x}}\right]^T . \mathbf{p} = \left[\dfrac{\partial \mathcal{H}}{\partial \mathbf{x}}\right]^T . (\mathcal{H}[\mathbf{x}] - \mathbf{y}_{obs}) \\ \mathbf{p}(T) = 0 \end{cases}$$

Gradient of the cost function

$$\nabla J = (\nabla_U J, \nabla_V J)^T = \left(-\left[\frac{\partial \mathcal{M}}{\partial \mathbf{x}}\right]^T . \mathbf{p}, -\mathbf{p}(0) \right)^T$$

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Variationnal Data Assimilation : The Optimality System

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Distributed Data Assimilation and Satellite Images

- Financial support : French Research Agency (ANR) : 2007 - 2009
- Leader : F.-X. Le Dimet (INRIA MOISE)
- Partners :
  - Team-project MOISE – INRIA Grenoble - Rhône-Alpes, France
  - Team-project CLIME – INRIA Paris - Rocquencourt, France
  - Institut de Mathématiques de Toulouse
  - Laboratory of Geophysical and Industrial Fluids (LEGI), Grenoble, France
  - Météo France – CNRM, Toulouse, France

http://addisa.gforge.inria.fr

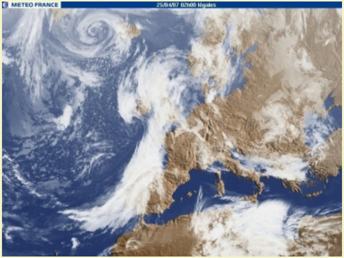
ADDISA project – Motivations

Sequence of images are widely available in meteorology, oceanography, hydrology, astrophysics, and even medicine. Historically they have been mainly used for a "by eye" analysis from experts in the field

The aim of the ADDISA project is to propose methods to use the information content of the image sequences in a data assimilation framework.

- Structure's dynamic into MÉTÉOSAT images (visible channel)

April 24, 2007, 20H00



April 25, 2007, 02h00

Source : Meteo France

- Sea Surface Temperature of the Black Sea (AVHRR infrared channel)

Source : NOAA

ADDISA project – Motivations

Meteorology : extreme events precursor

- Evolution of a Dry Intrusion

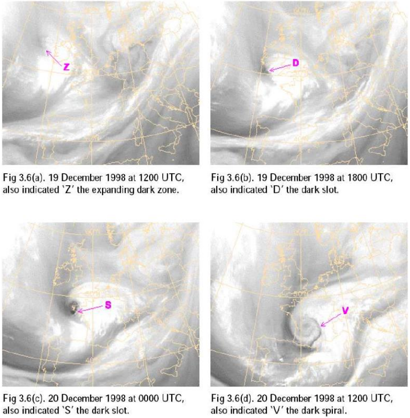


Fig 3.6(a). 19 December 1998 at 1200 UTC, also indicated 'Z' the expanding dark zone.

Fig 3.6(b). 19 December 1998 at 1800 UTC, also indicated 'D' the dark slot.

Fig 3.6(c). 20 December 1998 at 0000 UTC, also indicated 'S' the dark slot.

Fig 3.6(d). 20 December 1998 at 1200 UTC, also indicated 'V' the dark spiral.

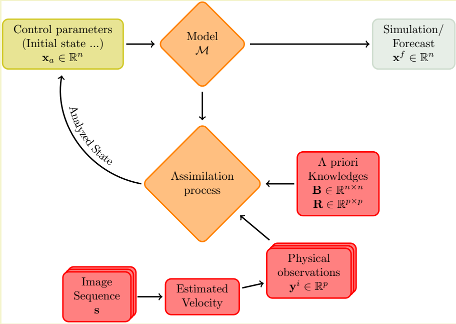
- Water Vapor Canal (MÉTÉOSAT)
- Evolution of a Dry Intrusion (24 hours)
- From a common anomaly to a cyclogenesis

(Santurette and Georgiev, 2005)

ADDISA project – Motivations

- Satellite image sequences show
  - A large amount of High Resolution Information
  - The evolution of some large scale structures such as vortices or fronts
  - Precursors of Extreme Events
- Currently, images and sequences of images are underused
- Application fields : Meteorology, oceanography, atmospheric sciences, hydrology, glaciology, medicine,. . .

Velocity fields can be estimated by some motion estimation techniques (e.g. optical flow, image model, optimal gradient,...) from sequences of images



- Estimated velocity fields are then injected as observations in the assimilation scheme

(G. Korotaev, E. Huot *et al.*, 2008), (D. Auroux and M. Masmoudi, 2006), (B. Horn and B. Schunck, 1981)

Pseudo observations technique

Velocity fields can be estimated by some motion estimation techniques (e.g. optical flow, image model, optimal gradient,...) from sequences of images

Basic idea: conservation of light $\frac{\partial I}{\partial t} + \nabla I \cdot \mathbf{v} = 0$

$$F(\mathbf{v}) = \frac{1}{2}\int_\Omega \| \frac{\partial I}{\partial t} + \nabla I \cdot \mathbf{v} \|^2 \, dxdy + \frac{1}{2}\lambda \|S(\mathbf{v})\|^2$$

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Example: velocity estimation by Image Model technique:

$$\begin{cases} \dfrac{\partial I}{\partial t} + \nabla I \cdot \mathbf{v} = 0 \\ \dfrac{\partial \mathbf{v}}{\partial t} = \mathcal{N}(\mathbf{v}), \quad I(0) = I_0, \quad \mathbf{v}(0) = \mathbf{v}_0 \end{cases}$$

$$F(\mathbf{v}_0) = \frac{1}{2}\int_\Omega \int_0^T [(I - I_{obs})^2 dxdydt + \frac{1}{2}\lambda \|S(\mathbf{v})\|^2$$

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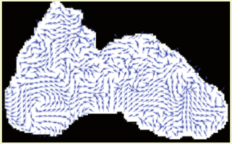

- Estimated velocity fields are then injected as observations in the assimilation scheme

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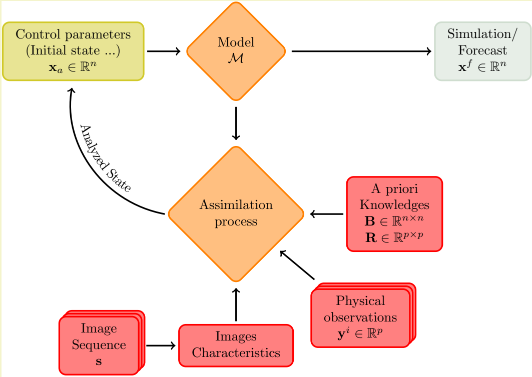
- Some characteristics are extracted from the image sequences (e.g. structures such as front, vortices. . .)



- Pseudo physical observations are avoided
- Characteristics are assimilated directly

$$J(x_0) = \underbrace{\int_0^T \|\mathbf{y} - \mathcal{H}[\mathcal{M}_t(x_0)]\|_{\mathcal{O}}^2 dt}_{\text{Classical term } J_o} + \int_0^T \| \underbrace{\mathcal{H}_{\mathcal{V}\to\mathcal{S}}[\mathbf{v}]}_{\text{Image to Structure Operator}} - \underbrace{\mathcal{H}_{\mathcal{X}\to\mathcal{S}}[\mathcal{M}_t(x_0)]}_{\text{Model to Structure Operator}} \|_{\mathcal{S}}^2 dt$$

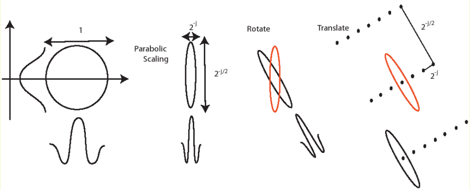
- $\mathcal{S}$ is the space of structures
- $\mathcal{H}_{\mathcal{V}\to\mathcal{S}}$ extracts structures from images
  - Frequential characteristics (multiscale transformations)
  - Geometric structures (snake, levelsets,. . .)
  - Qualitative characteristics
- $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ extracts structures from model outputs. Two ways :
  - $\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}}$ = composition of Synthetic Images Operator $\mathcal{H}_{\mathcal{X}\to\mathcal{V}}$ and the Image to Structures Operator (see following numerical results)
  - Defining directly $\mathcal{H}_{\mathcal{X}\to\mathcal{S}}$ without using synthetic images operator (work still in progress . . .)

$\mathcal{H}_{\mathcal{V}\to\mathcal{S}}(\mathbf{v})$ = Threshold of the Curvelet Transforms of the image $\mathbf{v}$

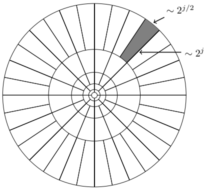
- Multi-scale, multi-orientation transformation with atoms indexed with a position parameter
- Decomposition : $\mathbf{v} = \sum_{j,k,l} \langle \mathbf{v}, \varphi_{j,l,k} \rangle \varphi_{j,l,k}$

  j : scale index
  l : orientation index (range depends of j)
  k : position index (range depends of j and l)

- Threshold : $\hat{\mathbf{v}}_m = \sum_{(j,k,l)\in E} \langle \mathbf{v}, \varphi_{j,l,k} \rangle \varphi_{j,l,k}$ $\quad \# E = m$



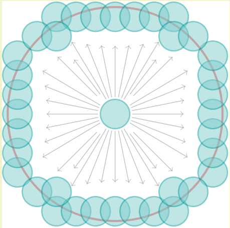
Principe of curvelets :
scaling, Rotations and translation



Spectral Partitioning of the frequency plane

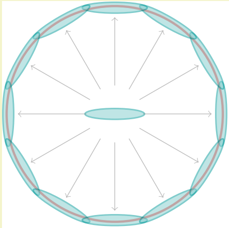
Curvelet transform is well adapted for images containing discontinuities

Wavelets



$$\|\mathbf{v} - \hat{\mathbf{v}}_m\| \approx m^{-1}$$

Curvelets



$$\|\mathbf{v} - \hat{\mathbf{v}}_m\| \approx C m^{-2} (\log m)^3$$

- For a fixed precision, curvelets need less coefficients to represents a smooth curve than wavelets
- Fast Discrete Curvelet Transform (FDCT) implementations runs in $O(n^2 \log n)$ for $n \times n$ cartesian arrays (`www.curvelet.org`)

(E. J. Candès and D. L. Donoho, 2004), (L. Demanet 2006)

Vortex motion in a rotating platform

Coriolis Platform
LEGI, Grenoble

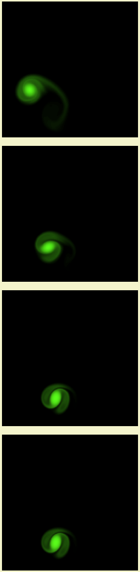
Isolated Vortex

< Experimentation

- J.-B. Flór (LEGI) and I. Eames, 2002

Numerical simulation >

- Shallow water model
- Advection of passive tracer



Evolution of state vector $x = (u, v, h)$: shallow-water model

$$\begin{cases} \partial_t u - u\partial_x u + v\partial_y u - fv + g\partial_x h + \mathcal{D}(u) & = & \mathcal{F}_u \\ \partial_t v + u\partial_x v + v\partial_y v + fu + g\partial_y h + \mathcal{D}(v) & = & \mathcal{F}_v \\ \partial_t h + \partial_x(hu) + \partial_y(hv) & = & 0 \end{cases}$$

Image to Structure Operator

$$\mathcal{H}_{\mathcal{I}\to\mathcal{S}}[\mathbf{I}] = \text{Threshold of FDCT}[\mathbf{I}] = \mathcal{T}(\text{FDCT}[\mathbf{I}])$$

Model to Structure Operator (through synthetic image generation

$$\mathcal{H}_{\mathcal{X}\to\mathcal{I}}[\mathbf{u},\mathbf{v},\mathbf{h}] = \mathbf{q} \quad : \quad \partial_t q + u\partial_x q + v\partial_y q - \nu_T \Delta q = 0$$

$$\mathcal{H}_{\mathcal{X}\to\mathcal{S}} = \mathcal{H}_{\mathcal{V}\to\mathcal{S}} \circ \mathcal{H}_{\mathcal{X}\to\mathcal{V}} \quad : \quad \text{Threshold of FDCT}[\mathbf{q}] = \mathcal{T}(\text{FDCT}[\mathbf{q}])$$

Cost function

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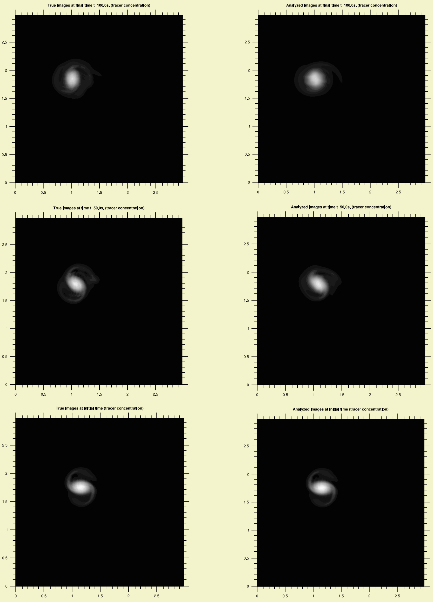
$t = 100$s.

$t = 50$s.

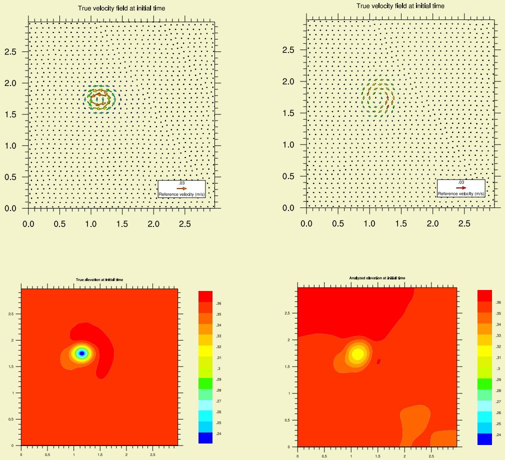
$t = 0$s.



True Images   Analyzed field

First Guess
System at rest
Assimilation window
100 seconds
(2000 time steps)
Observations
20 images
10% of the curvlets
(hard thresholding)

v



h

True fields — Analyzed fields

First Guess

System at rest

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(2000 time steps)

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Conclusions

- Image Sequences have a strong predictive potential
- Pseudo observation techniques retrieve observations of state variables from images (estimated velocity fields)
- Direct Assimilation of Image Sequences extract characteristics from image and model outputs in order to compare it in an appropriate space

Future works

- Definition of other characteristics
- Development of tools for characteristics extraction (e.g. 3D curvelet decomposition)
- Applications to other geophysical fluid flows structures (e.g. fronts)

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